

Answer Key: Linear Functions in Four Forms

Course: Algebra I (Grades 9–12) **Subject:** Mathematics **For teacher use only** Math

How to use this key

Answers are grouped by section and question number, with full worked steps. Accept any correct method that reaches the right value — alternate strategies (using different table rows, counting up on the graph, or reasoning from the context) are noted where they apply. Watch-for notes flag common misconceptions. A basic or graphing calculator is allowed, so grade the **reasoning and steps**, not just the arithmetic. Estimated grading time: ~6–8 min per packet.

Start Warm-Up: Retrieval ~1 min

1 Evaluate $f(x) = 2x + 3$ at $x = 4$, i.e. find $f(4)$.

$f(4) = 2 \times 4 + 3 = 8 + 3 = 11$. Full credit for showing the multiply-then-add order. *Watch-for:* a student who reads $f(4)$ as "f times 4" or who writes $2 \times (4 + 3) = 14$, adding before multiplying. Remind them $f(4)$ names the output at $x = 4$, and mx is done before adding b .

2 Slope through $(1, 5)$ and $(4, 11)$.

$m = (11 - 5) \div (4 - 1) = 6 \div 3 = 2$. Accept the points taken in either order as long as x -differences and y -differences use the **same** order: $(5 - 11) \div (1 - 4) = (-6) \div (-3) = 2$. *Watch-for:* reversing to run over rise, $(4 - 1) \div (11 - 5) = 3 \div 6 = 0.5$, which is wrong.

Build Study the Math ~1 min

3 Find $f(7)$ for $f(x) = 2x + 3$.

$f(7) = 2 \times 7 + 3 = 14 + 3 = 17$ (so 7 hours costs \$17). *Alternate:* extend the table by adding 2 each hour ($11 \rightarrow 13 \rightarrow 15 \rightarrow 17$). Both are correct.

Apply Use What You Know ~3–4 min

4 Match the four forms (repair cost, Table B).

4a. Using rows $(2, 55)$ and $(4, 85)$: $m = (85 - 55) \div (4 - 2) = 30 \div 2 = 15$ **dollars per part**. (Any two rows work: e.g., $(0, 25)$ and $(6, 115) \rightarrow 90 \div 6 = 15$.)

4b. y -intercept $b = 25$ (the cost at $x = 0$ parts, read straight from the table). Equation: **$g(x) = 15x + 25$** .

4c. The **slope** means the cost goes up **\$15 per part** (each part adds \$15). The **$y$ -intercept** means the **\$25 diagnostic fee** charged before any parts (at 0 parts the cost is already \$25).

4d. Solve $g(x) = 145$: $15x + 25 = 145 \rightarrow$ subtract 25: $15x = 120 \rightarrow$ divide by 15: $x = 8$ **parts**. *Check:* $15(8) + 25 = 120 + 25 = 145$. ✓ *Alternate:* $x = (145 - 25) \div 15 = 120 \div 15 = 8$.

4e. It is **NOT proportional**, because the y -intercept is 25, not 0 — the line does not pass through the origin $(0, 0)$. (A proportional function must have $b = 0$.)

5 Write the equation from a context (gym membership).

5a. Slope $m = 12$ (dollars per month) and y-intercept $b = 40$ (the \$40 sign-up fee, the cost at 0 months).

Equation: **$h(x) = 12x + 40$** .

5b. $h(9) = 12(9) + 40 = 108 + 40 = \mathbf{\$148}$ after 9 months. *Watch-for:* a student who writes $h(9) = 12 + 40(9)$ by swapping which number multiplies x — anchor that the **per-month rate** multiplies the months and the **sign-up fee** is the constant b .

6 Error analysis: a mis-found slope for Table B.

The mistake: The student **reversed rise and run** — they divided $\text{run} \div \text{rise}$ instead of $\text{rise} \div \text{run}$. Slope is the change in the output (rise) over the change in x (run), so the differences are flipped.

Correct work: $m = \text{rise} \div \text{run} = (85 - 55) \div (4 - 2) = 30 \div 2 = \mathbf{15 \text{ dollars per part}}$ (not 0.07).

Full credit requires (1) naming that they flipped rise and run (used run/rise), and (2) the correct slope of 15.

Sense-check to offer: 0.07 dollars per part would mean about 7¢ a part, which does not match the table jumping \$30 for every 2 parts.

Block Extra Applied Task block only • ~1–2 min

7 Real-world modeling: draining tank (block schedule).

7a. Starting amount $b = 60$ gallons; the tank **loses** 4 gallons per minute, so the rate of change is **-4**. Function:

$V(x) = -4x + 60$. *Watch-for:* a student who writes +4 and forgets the water is decreasing.

7b. $V(6) = -4(6) + 60 = -24 + 60 = \mathbf{36 \text{ gallons}}$ left after 6 minutes.

7c. Solve $V(x) = 0$: $-4x + 60 = 0 \rightarrow -4x = -60 \rightarrow x = \mathbf{15 \text{ minutes}}$. *Check:* $V(15) = -4(15) + 60 = -60 + 60 = 0$.

✓ **Interpretation:** the x -intercept (15, 0) is the moment the tank is **empty** — the input that makes the output 0.

Alternate: $x = (0 - 60) \div (-4) = (-60) \div (-4) = 15$.

Explain Justify Your Strategy (Q8) ~1–2 min

8 3 premium channels: Plan A $f(x) = 3x + 8$ vs. Plan B $g(x) = 5x$.

Model answer. Claim: For 3 premium channels, **Plan B** costs less; and **Plan B is the proportional one**.

Evidence: Plan A: $f(3) = 3(3) + 8 = 9 + 8 = \mathbf{\$17}$. Plan B: $g(3) = 5(3) = \mathbf{\$15}$. Since $\$15 < \17 , Plan B is cheaper for 3 channels. Reasoning: Plan B has y-intercept $b = 0$, so it passes through the origin and is **proportional** ($g(x) = 5x$). Plan A has y-intercept $b = 8$ (an \$8 base charge), so it is **not proportional**. Even though Plan A's slope (\$3/channel) is smaller than Plan B's (\$5/channel), Plan A's \$8 head start makes it cost more at 3 channels.

Note: Full credit requires both computed costs (\$17 and \$15), naming Plan B as cheaper, and identifying Plan B as proportional ($b = 0$). *Extension worth noting:* the two plans are equal when $3x + 8 = 5x \rightarrow 2x = 8 \rightarrow x = 4$ channels; beyond 4 channels Plan A becomes cheaper. Students are not required to find this, but it is a strong discussion point.

Justification scoring rubric (3 points)

Score	Claim	Evidence	Reasoning
3	Names Plan B as cheaper for 3 channels AND names Plan B as proportional.	Both costs computed correctly (\$17 and \$15) with the numbers shown.	Explains proportional means $b = 0$ and correctly links slope and y-intercept to the comparison.
2	Names Plan B as cheaper (proportional part missing or partly right).	One cost computed, or both with a minor arithmetic slip.	Reasoning present but incomplete or missing the y-intercept link.
1	A plan named with weak or no support.	Evidence largely missing or incorrect.	Little or flawed reasoning.

Score	Claim	Evidence	Reasoning
0	No/incorrect claim.	No evidence.	No reasoning.

Close ACE ~1 min

ACE Articulate / Connect / Extend.

Articulate: Accept any correct explanation: "Evaluating $f(x)$ means you are **given the input x** and you compute the **output** (e.g., $f(4) = 11$). **Solving** $f(x) = k$ means you are **given the output k** and you find the **input x** that produces it (e.g., $2x + 3 = 15 \rightarrow x = 6$). Evaluating gives an output; solving gives an input."

Connect: Valid items where function notation was used to find an output include Q1 ($f(4)$), Q3 ($f(7)$), Q5b ($h(9)$), Q7b ($V(6)$), or Q8 ($f(3)$, $g(3)$). Any of these earns credit if correctly named.

Extend: Any genuine non-proportional linear function with a starting amount and a steady rate — e.g., a taxi with a base fare plus a per-mile rate, a phone plan with a monthly fee plus a per-gigabyte charge, a savings account starting at some balance and growing by a fixed weekly deposit. Must have a nonzero starting amount ($b \neq 0$) to count as non-proportional.

Challenge Early Finisher (optional)

EF Write a context for $f(x) = 7x + 20$ (must be non-proportional).

(1) A sample situation: "A caterer charges a **\$20 setup fee** plus **\$7 per guest**. Let x = number of guests and $f(x)$ = total charge in dollars." Many correct stories exist (a \$20 flat delivery fee plus \$7 per item; a gift-card starting balance plus deposits, etc.).

(2) Meaning: The **slope 7** is the **rate of \$7 per guest** (each guest adds \$7). The **y-intercept 20** is the **\$20 starting amount / setup fee** charged before any guests (the cost at $x = 0$).

(3) Evaluate $f(5)$: $f(5) = 7(5) + 20 = 35 + 20 = \55 .

(4) Solve $f(x) = 90$: $7x + 20 = 90 \rightarrow 7x = 70 \rightarrow x = 10$ **guests**. *Check:* $7(10) + 20 = 90$. ✓

Full credit requires a clear x and $f(x)$, a story with a real starting amount (so it is non-proportional), a correct interpretation of 7 and 20, and the correct values 55 and 10.

Watch Common misconceptions

- **Slope vs. y-intercept.** Students may swap the two — reporting the starting amount as the rate, or the rate as the starting amount. In $g(x) = 15x + 25$, $m = 15$ is the per-part rate and $b = 25$ is the diagnostic fee. Anchor: the number multiplied by x is the slope; the number added on is the y-intercept (the output at $x = 0$).
- **Reversing rise and run.** The Q6 error: computing $\text{run} \div \text{rise}$ (giving ≈ 0.07) instead of $\text{rise} \div \text{run}$ (giving 15). Slope is **change in output over change in x** — always the y-difference on top. Subtract the x-values and y-values in the **same order**.
- **Confusing $f(x)$ with multiplication.** $f(4)$ does **not** mean "f times 4"; it names the **output** when the input is 4. Reinforce this on Q1, Q3, Q5b, Q7b, Q8. A student who "distributes" f across (4) has misread the notation.
- **Assuming all lines are proportional.** Students often treat any straight line as proportional. A linear function is proportional **only if it passes through (0, 0)**, i.e., $b = 0$. Figure 1 and Table B both have $b \neq 0$, so they are linear but **not** proportional (Q4e and Q8 check this).
- **Evaluate vs. solve.** Some students "solve" when asked to evaluate, or plug in the output where the input goes. Anchor: **given $x \rightarrow$ evaluate** (find output); **given output $\rightarrow$ solve** (find x). Q4d, Q7c, and the ACE Articulate item test this.
- **Dropping the negative rate.** In Q7, water is draining, so $m = -4$. A student who writes $V(x) = 4x + 60$ has missed that the output decreases. Encourage a check: after 15 minutes the tank should be empty, not overflowing.

- **Stopping at the wrong step when solving.** In Q4d/Q7c, after isolating the mx term a student may forget to divide by m . Encourage a check by substituting the answer back into the equation.

Teacher follow-up based on likely errors

If many students miss Q6, do a quick 5-minute practice labeling rise and run on two or three graphs, always keeping the output-difference on top. If Q4e or Q8 show the "all lines are proportional" idea, re-anchor that only a line through $(0, 0)$ is proportional ($b = 0$), and contrast it with a line that has a fee. The Q4c interpretations reveal who can attach meaning to slope and y-intercept in context. The Q4d and Q7c items show who can move from evaluating to **solving** $f(x) = k$, and Q8 shows who can compare two functions using both slope and y-intercept — all good warm-up discussions next class.

HS_ALG1_LinearFunctions_01 — Linear Functions in Four Forms Teacher Answer Key