

The Pythagorean Theorem & Distance on the Coordinate Plane

Course: Geometry (Grades 9–12) **Subject:** Mathematics **Time:** ~50 min standard · ~90 min block **Work mode:** Independent · pencil + packet Math

Start Here

This is a **paper math packet** about the **Pythagorean Theorem** and using it to find **distance** on the coordinate plane. In a **right triangle**, the two shorter sides (the **legs**) and the longest side (the **hypotenuse**) are connected by the rule $a^2 + b^2 = c^2$. You will use it to find a missing side, to measure the **distance between two plotted points**, to solve a **real-world** problem (a ladder), to **fix an error**, and to test whether a triangle is a **right triangle** (the **converse**). Work by yourself with a pencil. **A calculator is not required, but a basic or scientific calculator is allowed** if you have one. **Show your work** — every answer should show the squares you computed and the square root you took, not just a final number. Give both an **exact** answer (such as $\sqrt{52}$) and a **rounded** answer (to the nearest tenth) when a number is not a perfect square. Everything you need (formulas, a worked example, a labeled graph) is printed right here. If a question is hard, skip it, keep going, and come back. Circle or box your final answers.

Start Warm-Up: Retrieval 6 min

Bring back squaring, square roots, and reading points off a grid.

1Square and square-root. Find 5^2 and 12^2 , then add them. Finally, find $\sqrt{169}$. Show each step. (Remember: 5^2 means 5×5 , not 5×2 .)

2Read a horizontal and a vertical distance. A point starts at **(2, 3)** and a second point is at **(2, 8)**. How far apart are they **up-and-down** (the vertical distance)? Then a third point is at **(6, 3)**: how far is it from **(2, 3)** **left-and-right** (the horizontal distance)? Show the subtraction you used.

Build Study the Math 8–12 min

What you need to know (with formulas)

A **right triangle** has one **90° angle** (a square corner). The two sides that form the right angle are the **legs** (call their lengths **a** and **b**). The side **across from** the right angle is the **hypotenuse** (length **c**). The hypotenuse is **always the longest side** — that is how you can spot it.

The Pythagorean Theorem. For any right triangle:

$$a^2 + b^2 = c^2$$

The two legs squared and added equal the hypotenuse squared. Use it two ways:

- **Missing hypotenuse (c):** add the squares of the legs, then take the square root: $c = \sqrt{a^2 + b^2}$.
- **Missing leg (a or b):** the hypotenuse is already known, so **subtract**: $a = \sqrt{c^2 - b^2}$. (Square the hypotenuse, subtract the known leg's square, then take the square root.)

The Distance Formula (an application). To find the distance between two points (x_1, y_1) and (x_2, y_2) on a grid, make a right triangle: the horizontal leg is the change in x and the vertical leg is the change in y. Then apply the theorem:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This is just $a^2 + b^2 = c^2$ where $a = (x_2 - x_1)$, $b = (y_2 - y_1)$, and $d = c$. Subtracting in the other order is fine because squaring makes it positive either way.

The Converse (a right-triangle test). The theorem also works *backward*. If the three side lengths of a triangle make $a^2 + b^2 = c^2$ true (with c the **longest** side), then the triangle **is a right triangle**. If $a^2 + b^2 \neq c^2$, it is **not** a right triangle. To use it: square all three sides, add the squares of the two shorter sides, and compare that sum to the square of the longest side.

Tip: when $a^2 + b^2$ is not a perfect square, the exact answer is a square root (like $\sqrt{52}$). Leave it as a root for the **exact** value, and also give a **rounded** decimal ($\sqrt{52} \approx 7.2$) to the nearest tenth.

Worked Example — a 6-8-10 right triangle on the grid

Look at Figure 1. The right angle is at **B(7, 1)**. One leg runs from **A(1, 1)** to **B(7, 1)**; the other leg runs from **B(7, 1)** to **C(7, 9)**. The hypotenuse is the slanted side from **A(1, 1)** to **C(7, 9)** — it is across from the right angle and is the longest side.

Find the legs by counting boxes: leg **a** = **AB** = **7 - 1 = 6** units (horizontal); leg **b** = **BC** = **9 - 1 = 8** units (vertical).

Find the hypotenuse c with the theorem:

$$a^2 + b^2 = c^2 \rightarrow 6^2 + 8^2 = c^2 \rightarrow 36 + 64 = c^2 \rightarrow 100 = c^2$$

$c = \sqrt{100} = \mathbf{10}$. So the hypotenuse AC is **10 units**. (Exact and rounded agree here because 100 is a perfect square.)

Same answer using the distance formula for A(1, 1) and C(7, 9): $d = \sqrt{(7 - 1)^2 + (9 - 1)^2} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = \mathbf{10}$. ✓

Check the converse: the sides are 6, 8, 10, and 10 is longest. $6^2 + 8^2 = 36 + 64 = 100$, and $10^2 = 100$. Since $100 = 100$, the triangle **is** a right triangle. ✓

The three sides of triangle ABC (right angle at B).

Side	From → To	Type	Length (units)
AB	(1, 1) → (7, 1)	leg a (horizontal)	6
BC	(7, 1) → (7, 9)	leg b (vertical)	8
AC	(1, 1) → (7, 9)	hypotenuse c	10

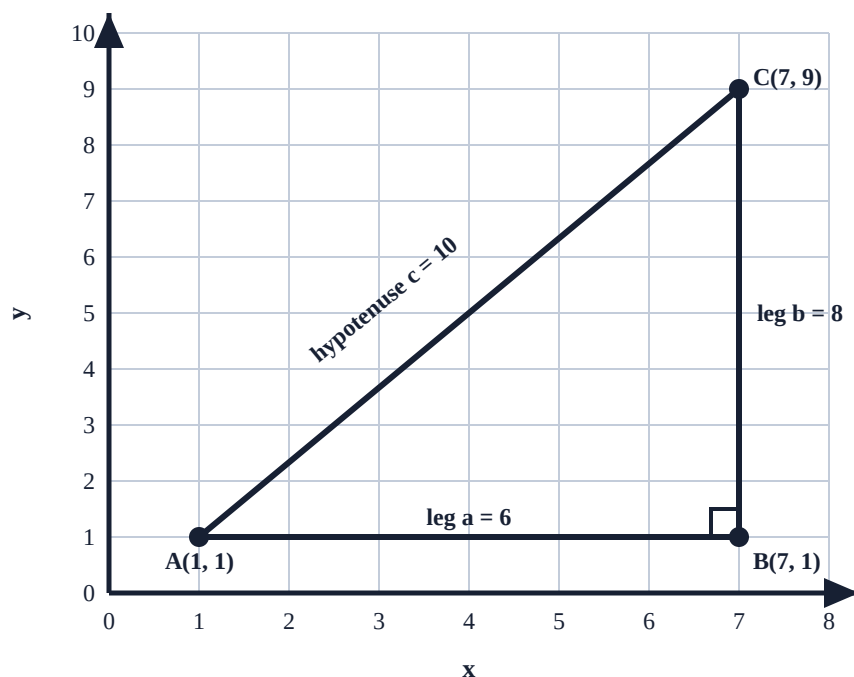


Figure 1. A 6-8-10 right triangle with vertices **A(1, 1)**, **B(7, 1)**, and **C(7, 9)**. The right angle (small square) is at B. Leg **a** = **AB** = **6**, leg **b** = **BC** = **8**, and the hypotenuse **c** = **AC** = **10**, because $6^2 + 8^2 = 36 + 64 = 100$ and $\sqrt{100} = 10$. The triangle and labels are drawn in black with text so the figure reads clearly in grayscale.

3Using Figure 1, suppose the vertical leg **BC** were **12** instead of 8 (with the horizontal leg **AB** still **6**). Use $a^2 + b^2 = c^2$ to find the new hypotenuse. Give the **exact** answer (a square root if needed) and the answer **rounded to the nearest tenth**. Show your work.

Apply Use What You Know 20–26 min

Show the squares and the square root in every item. A calculator is allowed. When the answer is not a perfect square, give both the exact root and the rounded decimal.

4Find a missing side.

4a. Missing hypotenuse. A right triangle has legs **a** = **9** and **b** = **12**. Find the hypotenuse **c**. Use $a^2 + b^2 = c^2$ and show every step. (This one comes out to a whole number.)

4b. Missing leg. A different right triangle has one leg **b** = **5** and hypotenuse **c** = **13**. Find the other leg **a**. (Careful: the hypotenuse is known, so you must **subtract**: $a^2 = c^2 - b^2$.) Show your steps.

5**Distance between two plotted points.** On a coordinate grid, point **P** is at **(1, 2)** and point **Q** is at **(7, 10)**. Find the **distance PQ**. Use the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$; show the horizontal change, the vertical

change, both squares, the sum, and the square root.

6Real-world application (a ladder). A **17-foot** ladder leans against a wall. Its base is **8 feet** from the bottom of the wall. **How high up the wall does the ladder reach?** The wall and ground make the right angle, the ground distance and the wall height are the **legs**, and the ladder is the **hypotenuse**. Draw a quick right triangle, label it, and solve. Show your work.

7Error analysis (added instead of using the theorem). A student found the hypotenuse of a right triangle with legs **a = 6** and **b = 8** and wrote the work shown. **Find the mistake, explain it, and give the correct hypotenuse.**

Student's work (legs 6 and 8):

$c = a + b = 6 + 8 = 14$. So the hypotenuse is 14.

What is wrong, and what is the correct hypotenuse? Fix it in the box, showing the correct use of $a^2 + b^2 = c^2$.

8Use the converse to test a triangle. A triangle has side lengths **10, 24, and 26**. **Is it a right triangle?** Identify the **longest** side, square all three sides, and check whether $a^2 + b^2 = c^2$. State your conclusion clearly (yes or no) and why.

Block Extra Applied Task block only • ~12–15 min

If your class is on a ~90-minute block schedule, complete Item 9. (On a standard ~50-minute period, this item is optional.)

9Multi-step composite distance. A hiker walks a path with three stops plotted on a grid (each unit = 1 kilometer): **R(0, 0)**, then **S(3, 4)**, then **T(10, 4)**. She walks a straight segment from R to S, then a straight segment from S to T.

9a. Find the length of segment **RS** using the distance formula. Show your work.

9b. Find the length of segment **ST** using the distance formula. Show your work.

9c. Find the **total distance walked** ($RS + ST$). Then find the **straight-line distance** from R directly to T (the distance formula for R to T). Which is longer, the path or the straight line? Explain in one sentence why that makes sense.

Explain Justify Your Strategy 6–10 min

Question 10. A triangle has side lengths **7, 9, and 12**. A classmate says, "It must be a right triangle because it has three different sides." **Is the triangle a right triangle? Decide, and justify your answer** using the converse of the Pythagorean Theorem. Write a **Claim** (your yes/no answer), **Evidence** (the squaring and adding you computed), and **Reasoning** (why that computation proves your claim, and why the classmate's reason is not enough).

Sentence stems you may use: "The triangle is / is not a right triangle because..." · "The longest side is ____, so $c = ___.$ " · "I found $a^2 + b^2 = ___$ and $c^2 = ___.$ " · "Since $a^2 + b^2 ___ c^2$, the converse tells me..." · "Having three different sides does not prove a right angle because..."

Claim (your yes/no answer)

Evidence (the numbers you computed)

Reasoning (why the computation proves it)

Close ACE Wrap-Up 5 min

Articulate

In your own words, explain how you can tell which side of a right triangle is the **hypotenuse**, and why you must **square-root** at the end instead of stopping at c^2 .

Connect

Point to one item in this packet where the **distance formula** was really just $a^2 + b^2 = c^2$ in disguise. Give its number and name the legs.

Extend

Give a **new** real-life situation (not the ladder) where you could use the Pythagorean Theorem to find a distance you cannot measure directly.

Continue Early Finisher optional

If you finish early (a challenge — no new materials needed):

Find a Pythagorean triple of your own. A **Pythagorean triple** is three whole numbers a, b, c with $a^2 + b^2 = c^2$ (like 3-4-5 or 6-8-10). On the back of this page: (1) start with the triple **3-4-5** and **multiply all three numbers by the same whole number** (try 2, then 3) to make two **new** triples; (2) **prove** each new set really works by showing $a^2 + b^2 = c^2$; and (3) explain in one sentence **why** multiplying a triple by the same number always gives another right triangle. Then use the **converse** to show that **5-6-8** is **not** a right triangle.

Turn in: Hand in this whole packet with your name, class period, and date filled in. Make sure items 1–8 and 10 and the ACE box are answered **with your work shown** (item 9 too if you are on a block schedule), including

both exact and rounded answers where needed. The early-finisher challenge is optional but turn it in too if you did it.

HS_GEOM_Pythagorean_01 — The Pythagorean Theorem & Distance on the Coordinate Plane Student Packet