

Quadratic Functions: Graphs, Zeros, and the Vertex

Course: Algebra II (Grades 9–12) **Subject:** Mathematics **Time:** ~50 min standard · ~90 min block **Work mode:** Independent · pencil + packet Math

Start Here

This is a **paper math packet** about **quadratic functions** — functions whose graphs are **parabolas** (U-shaped curves). You will read a quadratic written in **standard form** $y = ax^2 + bx + c$, find its **vertex** (the turning point) and its **axis of symmetry**, find its **zeros (roots)** by **factoring**, and **interpret the vertex** as a maximum or minimum in a real situation. Work by yourself with a pencil. **A calculator is not required, but a graphing or scientific calculator is allowed** if you have one. **Show your work** — every answer should show the numbers and steps you used, not just a final number. Everything you need (formulas, a worked example, a graph) is printed right here. If a question is hard, skip it, keep going, and come back. Circle or box your final answers.

Start Warm-Up: Retrieval 6 min

Bring back what you already know about evaluating and factoring.

1Evaluate a quadratic. A function is defined by $f(x) = x^2 - 4x + 3$. Find $f(0)$ and $f(2)$. Show the squaring, the multiplication, and the addition you used. (Remember: square the input first, then multiply, then combine.)

2Factor and use the Zero Product Property. Factor $x^2 + 5x + 6$ into two binomials (find two numbers that multiply to 6 and add to 5). Then find the two values of x that make the product equal 0.

Build Study the Math 8–12 min

What you need to know (with formulas)

A **quadratic function** has a graph that is a **parabola** — a symmetric U-shaped curve. Its **standard form** is:

$$y = ax^2 + bx + c \quad (a \neq 0)$$

The number **a** controls the opening. If $a > 0$ the parabola **opens up** and its vertex is the **lowest** point (a **minimum**). If $a < 0$ it **opens down** and its vertex is the **highest** point (a **maximum**). The number **c** is the **y-intercept** — the output when $x = 0$, the point $(0, c)$.

Vertex. The **vertex** is the turning point of the parabola. Its x -coordinate is found from a and b :

$$x_{\text{vertex}} = -b \div (2a)$$

Then substitute that x back into the function to get the y -coordinate. The vertex is the point $(x_{\text{vertex}}, y_{\text{vertex}})$.

Axis of symmetry. The parabola is a mirror image across the **vertical line** through the vertex:

$$x = -b \div (2a) \quad (\text{a vertical line})$$

Two points with the **same output** sit at the **same distance** on each side of this line. That is why the two zeros (below) are equally spaced around the axis.

Zeros (roots). The **zeros** (also called **roots** or **x-intercepts**) are the x-values where the output is **0** — where the parabola crosses the x-axis. To find them by **factoring**, set $y = 0$ and factor:

$$ax^2 + bx + c = 0 \rightarrow (x - r_1)(x - r_2) = 0 \rightarrow x = r_1 \text{ or } x = r_2$$

This uses the **Zero Product Property**: if two factors multiply to 0, then at least one factor must be 0. Set **each factor equal to 0** and solve.

Vertex as max or min in context. In a real story, the vertex often answers "how high" or "how much." For a thrown object, the vertex is the **greatest height** (a maximum): the x-coordinate tells **when** and the y-coordinate tells **how high**. For an area or cost problem the vertex can be a **maximum area** or a **minimum cost**.

Tip: the axis of symmetry $x = -b \div (2a)$ and the vertex share the **same** x-value. Find x first; the axis is that vertical line and the vertex is that point on the curve.

Worked Example — $y = x^2 - 4x + 3$

Here $a = 1$, $b = -4$, $c = 3$. Because $a = 1 > 0$, the parabola **opens up**, so the vertex is a **minimum**. The **y-intercept** is $c = 3$, the point $(0, 3)$.

Axis of symmetry / vertex x: $x = -b \div (2a) = -(-4) \div (2 \cdot 1) = 4 \div 2 = 2$. So the axis of symmetry is the vertical line $x = 2$.

Vertex y: substitute $x = 2$: $f(2) = (2)^2 - 4(2) + 3 = 4 - 8 + 3 = -1$. The **vertex is $(2, -1)$** — the lowest point.

Zeros by factoring: set $x^2 - 4x + 3 = 0$. Two numbers that multiply to +3 and add to -4 are -1 and -3, so it factors as $(x - 1)(x - 3) = 0$. By the Zero Product Property, $x - 1 = 0$ or $x - 3 = 0$, giving $x = 1$ or $x = 3$. The parabola crosses the x-axis at $(1, 0)$ and $(3, 0)$.

Symmetry check: the two zeros 1 and 3 are the same distance (1 unit) on each side of the axis $x = 2$. The midpoint of the zeros, $(1 + 3) \div 2 = 2$, is the axis. ✓

Outputs of $f(x) = x^2 - 4x$
+ 3 (notice the symmetry
around $x = 2$).

x	f(x)	Note
0	3	y-intercept $(0, 3)$
1	0	zero / x-intercept
2	-1	vertex (minimum)
3	0	zero / x-intercept
4	3	mirror of $(0, 3)$

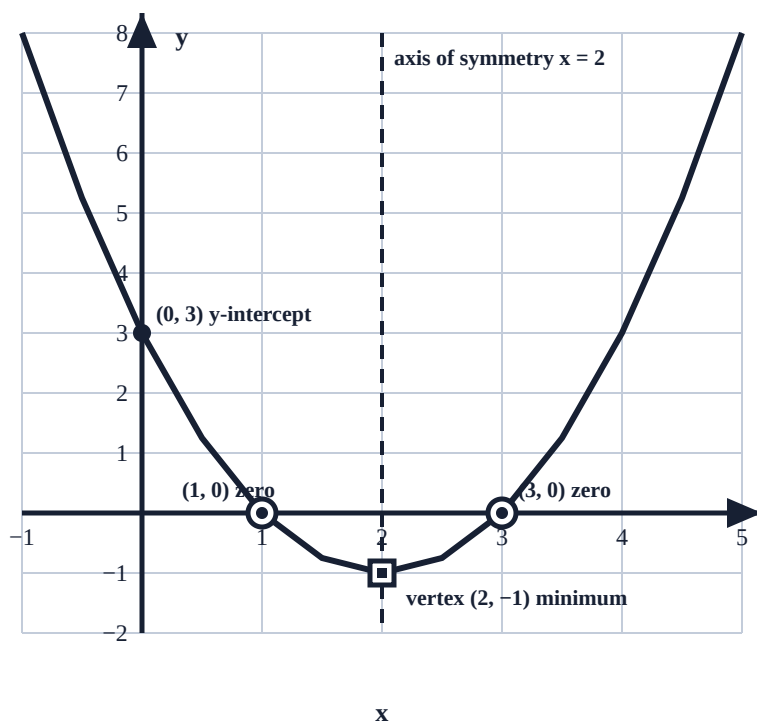


Figure 1. The graph of $y = x^2 - 4x + 3$. The parabola **opens up** ($a = 1 > 0$), so its vertex **(2, -1)** is a **minimum**. The dashed vertical line is the **axis of symmetry $x = 2$** . The curve crosses the x-axis at the two zeros **(1, 0)** and **(3, 0)** and crosses the y-axis at **(0, 3)**. Each plotted point satisfies the equation (for example $f(1) = 0$, $f(2) = -1$, $f(3) = 0$). The curve, points, and labels are drawn in black with text so the figure reads clearly in grayscale.

3Using the worked example graph, find **$f(4)$** for $f(x) = x^2 - 4x + 3$, and explain in one sentence why $f(4)$ has the **same** value as $f(0)$. (Hint: use the axis of symmetry $x = 2$.)

Apply Use What You Know 20–26 min

Show the numbers you use in every item. A calculator is allowed.

4**Read a graph.** A parabola is described below. It opens up and passes through the points $(-1, 0)$, $(0, -3)$, $(1, -4)$, $(2, -3)$, and $(3, 0)$. Use the description to answer 4a–4d.

Table
for Item
4.

Points
on the
parabola
(opens
up).

x	y
-1	0
0	-3
1	-4
2	-3
3	0

4a. Name the two **zeros (x-intercepts)** — the x-values where $y = 0$.

4b. Give the **vertex** (the lowest point in the table) as an ordered pair, and state whether it is a **maximum** or a **minimum**.

4c. Give the **axis of symmetry** as an equation $x = \underline{\hspace{2cm}}$. Explain in one sentence how the two zeros show you the axis.

4d. What is the **y-intercept** (the value of y when $x = 0$)?

5Factor to find the roots. Solve $x^2 - 7x + 10 = 0$ by factoring. Show the two numbers you found (they multiply to 10 and add to -7), the factored form, and each root from the Zero Product Property. Then state the **axis of symmetry** (halfway between the roots).

6Match a quadratic to a context. A soccer ball is kicked and its height in feet after t seconds is $h(t) = -16t^2 + 32t$ (here $a = -16$, $b = 32$, $c = 0$).

6a. Does this parabola open **up** or **down**? Is the vertex a **maximum** or a **minimum**? Answer in one sentence, using the sign of a .

6b. Find the **time** of the vertex using $t = -b \div (2a)$, then find the **greatest height** by evaluating h at that time. Show your steps and interpret the vertex in the story (when is the ball highest, and how high?).

7Error analysis (a sign error in factoring). A student solved $x^2 - x - 6 = 0$ and wrote the work shown. **Find the mistake, explain it, and give the correct roots.**

Student's work:

$$x^2 - x - 6 = (x - 2)(x - 3) = 0$$

So $x - 2 = 0$ or $x - 3 = 0$, giving $x = 2$ or $x = 3$.

What is wrong, and what are the correct roots? Fix it in the box.

Block Extra Applied Task block only • ~12–15 min

If your class is on a ~90-minute block schedule, complete Item 8. (On a standard ~50-minute period, this item is optional.)

8Vertex form and a second model. The **vertex form** of a quadratic is $y = a(x - h)^2 + k$, where the vertex is the point **(h, k)** and the axis of symmetry is $x = h$. This form shows the vertex directly, with no calculation needed.

8a. A parabola is given by $y = 2(x - 3)^2 - 5$. Write its **vertex** (h , k) and its **axis of symmetry**. Does it open up or down, and is the vertex a max or a min?

8b. Area modeling. A rancher has **40 feet** of fence to enclose a rectangular pen against a barn wall (the wall is one side, so the fence covers the other **three** sides). If the width is x , the two widths and one length use $2x + \text{length} = 40$, so $\text{length} = 40 - 2x$, and the area is $A(x) = x(40 - 2x) = -2x^2 + 40x$. Find the width x at the **vertex** (use $x = -b \div (2a)$), then the **maximum area**. Show your steps and interpret the vertex (what width gives the biggest pen, and how big is it?).

Explain Justify Your Strategy 6–10 min

Question 9. A classmate says, "For the quadratic $y = x^2 - 6x + 8$, the vertex is at $x = 4$ because the two roots are 2 and 4." **Is the classmate right? Find the true roots and the true vertex, and decide.** Write a **Claim** (your answer), **Evidence** (the roots you found by factoring and the vertex x from $-b \div (2a)$), and **Reasoning** (why the axis of symmetry sits **halfway between** the roots, not **at** a root).

Sentence stems you may use: "The classmate is ___ because..." · "Factoring $x^2 - 6x + 8$ gives... so the roots are..." · "The axis of symmetry is $x = -b \div (2a) = \dots$ " · "The vertex sits halfway between the roots because..." · "A root is where $y = 0$, but the vertex is where the curve turns, so..."

Claim (your answer / strategy)

Evidence (the numbers you used)

Reasoning (why the strategy works)

Close ACE Wrap-Up 5 min

Articulate

In your own words, explain the difference between a **zero (root)** of a quadratic and its **vertex**. Which one is where the curve crosses the x -axis, and which one is where the curve turns?

Connect

Point to one item in this packet where you found a **vertex** and used it as a **maximum or minimum** in a real context. Give its number.

Extend

Give a **new** real-life example where finding a **vertex** answers a useful question (a maximum or a minimum) that was *not* in this packet.

Continue Early Finisher optional

If you finish early (a challenge — no new materials needed):

Build your own quadratic. On the back of this page: (1) choose two whole-number roots you like (for example 2 and 5) and write a factored quadratic $(x - r_1)(x - r_2)$; (2) **multiply it out** into standard form $y = x^2 + bx + c$; (3) find the **axis of symmetry** and the **vertex**; and (4) make a small table of 5 outputs and sketch the parabola,

marking the vertex, the axis, and both zeros. Check that your two roots are the same distance on each side of your axis.

Turn in: Hand in this whole packet with your name, class period, and date filled in. Make sure items 1–7 and 9 and the ACE box are answered **with your work shown** (item 8 too if you are on a block schedule). The early-finisher challenge is optional but turn it in too if you did it.

HS_ALG2_Quadratics_01 — Quadratic Functions: Graphs, Zeros, and the Vertex Student Packet