

Linear Functions in Four Forms

Course: Algebra I (Grades 9–12) **Subject:** Mathematics **Time:** ~50 min standard · ~90 min block **Work mode:** Independent · pencil + packet Math

Start Here

This is a **paper math packet** about **linear functions** — functions whose graphs are straight lines. You will connect the **same** function shown four ways: a **table**, a **graph**, an **equation** in **function notation $f(x)$** , and a **real-world context**. You will also **write** equations from information and **solve** equations such as $f(x) = a$. Work by yourself with a pencil. **A calculator is not required, but a basic or graphing calculator is allowed** if you have one. **Show your work** — every answer should show the numbers and steps you used, not just a final number. Everything you need (formulas, a worked example, a graph) is printed right here. If a question is hard, skip it, keep going, and come back. Circle or box your final answers.

Start Warm-Up: Retrieval 6 min

Bring back what you already know about evaluating a function and finding slope.

1Evaluate a function. A function is defined by $f(x) = 2x + 3$. Find $f(4)$. Show the multiplication and the addition you used. (Remember: $f(4)$ means "the output when the input x is 4" — it does *not* mean f times 4.)

2Find slope from two points. A line passes through the points **(1, 5)** and **(4, 11)**. Find the **slope m** . Show $m = (y_2 - y_1) \div (x_2 - x_1)$ with your numbers.

Build Study the Math 8–12 min

What you need to know (with formulas)

A **linear function** has a graph that is a **straight line**. Its output changes by the **same amount** for each equal step of the input.

Function notation. We write $f(x)$ ("f of x") for the **output** of the function at input x . So $f(4)$ is the output when $x = 4$. The notation $f(x)$ is *not* multiplication — it names the output, not "f times x."

The **slope** (written m) is the **rate of change**: how much the output changes for each 1 that x increases. From two points (x_1, y_1) and (x_2, y_2) :

$$m = \text{rise} \div \text{run} = (y_2 - y_1) \div (x_2 - x_1)$$

The **y-intercept** (written b) is the output when $x = 0$ — the point **(0, b)** where the line crosses the vertical axis. The **x-intercept** is where the line crosses the horizontal axis; it is the value of x that makes the output **0** (solve $f(x) = 0$). Slope and y-intercept give the **slope-intercept form**:

$$y = mx + b \quad \text{or, in function notation,} \quad f(x) = mx + b$$

These say the same thing: y and $f(x)$ both name the output. Use "y" when you graph an ordered pair (x, y) ; use " $f(x)$ " when you want to talk about the function's output at a named input.

A linear function can be shown **four ways**, and all four match the same m and b :

- **Table:** as x goes up by a fixed step, the output goes up (or down) by a fixed step. $m = (\text{change in output}) \div (\text{change in } x)$. The row where $x = 0$ gives b .
- **Graph:** a straight line. It crosses the y -axis at $(0, b)$ and rises m units for each 1 unit right.
- **Equation:** $f(x) = mx + b$.
- **Context:** a real situation with a **starting amount** (b) and a steady **rate of change** (m) per one unit.

Solving $f(x) = k$. To find the input x that gives a chosen output k , replace $f(x)$ with k and solve the linear equation:

$$mx + b = k \rightarrow mx = k - b \rightarrow x = (k - b) \div m$$

Tip: "evaluate" means you are **given x** and you find the output. "Solve" means you are **given the output** and you find x . They are opposite directions.

Worked Example — one function, four forms

A tutoring service charges a flat booking fee plus a steady hourly rate. The cost **starts at \$3** (a booking fee) and rises **\$2 for every hour**. Let x = hours and let $f(x)$ = total cost in dollars. Because there is a **starting amount of 3**, this line does **not** pass through the origin.

Slope (rate of change): $m = 2$ dollars per hour. **y-intercept:** $b = 3$ dollars (the cost at 0 hours).

Equation (function notation): $f(x) = 2x + 3$.

Check the slope from two table rows: using $(1, 5)$ and $(3, 9)$, $m = (9 - 5) \div (3 - 1) = 4 \div 2 = 2$. ✓

Evaluate: $f(4) = 2(4) + 3 = 8 + 3 = 11$, so 4 hours costs \$11.

Solve $f(x) = 15$: $2x + 3 = 15 \rightarrow 2x = 12 \rightarrow x = 6$, so \$15 buys 6 hours.

Cost over hours ($f(x) = 2x + 3$).

Hours x Cost $f(x)$ (dollars) Change in output

0	3	— (start)
1	5	+2
2	7	+2
3	9	+2
4	11	+2

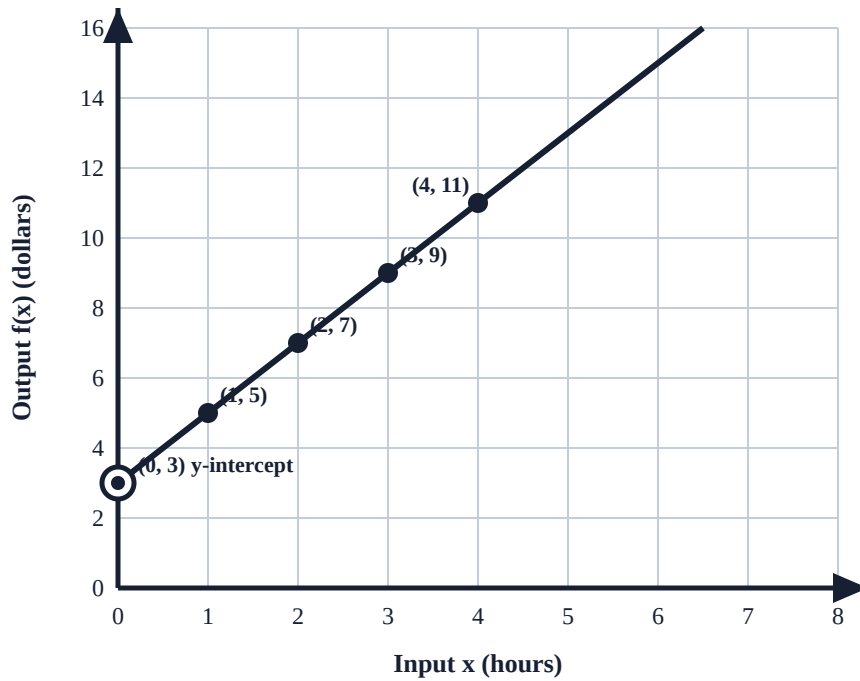


Figure 1. The graph of $f(x) = 2x + 3$. It is a straight line that crosses the vertical axis at **(0, 3)** — not at the origin — so it is **linear but not proportional**. Each printed point (1, 5), (2, 7), (3, 9), (4, 11) matches a table row and rises 2 units for every 1 unit right. The line and points are drawn in black with text labels so the figure reads clearly in grayscale.

3Using the worked example, find **$f(7)$** (the cost at 7 hours). Use $f(x) = 2x + 3$ and show your work.

Apply Use What You Know 20–26 min

Show the numbers you use in every item. A calculator is allowed.

4**Match the four forms.** A phone-repair shop charges a diagnostic fee plus a fixed price for each part used. The **same** linear function is shown below as a table, a graph description, an equation, and a context. Study Table B, then answer 4a–4e.

Table B. Total repair cost as a function of parts used.

Parts x Total cost $g(x)$ (dollars)

0	25
2	55
4	85
6	115

Context: "A repair shop charges a one-time **\$25 diagnostic fee**, then **\$15 for each part** used. Total cost = diagnostic fee + \$15 times the number of parts."

4a. Find the **slope** m (dollars per part). Use two rows of the table and show $m = (\text{change in output}) \div (\text{change in } x)$.

4b. Find the **y-intercept** b (the cost at 0 parts), then write the equation in function notation, $g(x) = mx + b$.

4c. Interpret in context. What does the **slope** mean, and what does the **y-intercept** mean? Use the words "per part" and "diagnostic fee."

4d. Solve $g(x) = 145$. A repair cost a total of **\$145**. How many **parts** were used? Set up your equation and solve for x , showing every step.

4e. Is this function **proportional** or **not proportional**? Explain using the y -intercept in one sentence.

5 Write the equation from a context. A gym membership starts with a **\$40 sign-up fee** and then costs **\$12 per month**. Let x = number of months and let $h(x)$ = total cost in dollars.

5a. Identify the **slope** m and the **y-intercept** b from the context, then write $h(x) = mx + b$.

5b. Use your equation to find $h(9)$, the total cost after 9 months. Show your work.

6 Error analysis (a mis-found slope). A student found the slope for Table B and wrote the work shown. **Find the mistake, explain it, and give the correct slope.**

Student's work (using the rows (2, 55) and (4, 85)):

$$m = \text{run} \div \text{rise} = (4 - 2) \div (85 - 55)$$

$$m = 2 \div 30 \approx 0.07. \text{ So the slope is about 0.07 dollars per part.}$$

What is wrong, and what is the correct slope? Fix it in the box.

Block Extra Applied Task block only • ~12–15 min

If your class is on a ~90-minute block schedule, complete Item 7. (On a standard ~50-minute period, this item is optional.)

7 Real-world modeling. A water tank begins with **60 gallons** and is **draining at 4 gallons per minute**. Let x = minutes and let $V(x)$ = gallons of water left in the tank.

7a. Write the function $V(x) = mx + b$. (Careful: the water is going *down*, so the rate of change is **negative**.)

7b. Find $V(6)$ — how much water is left after 6 minutes? Show your work.

7c. Solve $V(x) = 0$ (the x-intercept). After how many minutes is the tank empty? Show your steps, then interpret what the x-intercept means in this story.

Explain Justify Your Strategy 6–10 min

Question 8. Two streaming services charge a monthly bill. **Plan A:** $f(x) = 3x + 8$ (an \$8 base charge plus \$3 per premium channel). **Plan B:** $g(x) = 5x$ (just \$5 per premium channel, no base charge). **For 3 premium channels, which plan costs less, and why?** Also state which plan is **proportional**. Write a **Claim** (your answer), **Evidence** (the numbers you calculated), and **Reasoning** (why your strategy works, using slope and y-intercept).

Sentence stems you may use: "For 3 channels, Plan ____ costs less because..." · "I found Plan A's cost by... and Plan B's cost by..." · "Plan B is proportional because its y-intercept is..." · "The slope tells me... and the y-intercept tells me..."

Claim (your answer / strategy)

Evidence (the numbers you used)

Reasoning (why the strategy works)

Close ACE Wrap-Up 5 min

Articulate

In your own words, explain the difference between **evaluating** $f(x)$ at a value and **solving** $f(x) = k$. Which one gives an output, and which gives an input?

Connect

Point to one item in this packet where you used **function notation $f(x)$** to find an output. Give its number.

Extend

Give a **new** real-life example of a **non-proportional** linear function (a starting amount plus a steady rate) that was *not* in this packet.

Continue Early Finisher optional

If you finish early (a challenge — no new materials needed):

Write a context for a function. Here is a function with no story: **$f(x) = 7x + 20$** . On the back of this page: (1) invent a **real-world situation** that this function could describe, clearly telling what **x** and **$f(x)$** stand for; (2) explain what the **slope 7** and the **y-intercept 20** mean in your story; (3) **evaluate $f(5)$** , showing your work; and (4) **solve $f(x) = 90$** , showing your steps. Make sure your story is **non-proportional** (it must have a starting amount).

Turn in: Hand in this whole packet with your name, class period, and date filled in. Make sure items 1–6 and 8 and the ACE box are answered **with your work shown** (item 7 too if you are on a block schedule). The early-finisher challenge is optional but turn it in too if you did it.

HS_ALG1_LinearFunctions_01 — Linear Functions in Four Forms Student Packet