

Substitute Guide: The Pythagorean Theorem & Distance

Course: Geometry (Grades 9–12) **Subject:** Mathematics **Time:** ~50 min standard · ~90 min block Math

Learning goal (plain language)

Students practice the **Pythagorean Theorem** — $a^2 + b^2 = c^2$ for right triangles — and use it to find a **missing leg or hypotenuse**, to measure the **distance between two points** on a grid (the distance formula), to solve a **real-world ladder problem**, and to test whether a triangle is a **right triangle** using the **converse**. **You do not need a math background to run this.** Every formula, a worked example, and a labeled graph are printed in the student packet. Students **show their work**; you do not need to teach the method — the packet does the teaching. Your job is to hand it out, read the start script, keep the room on task, and collect the packets.

Standards alignment

Framework: 19 TAC §111.41 (Geometry). Knowledge & skills for right-triangle relationships and coordinate geometry — the Pythagorean Theorem and its converse, and finding distance on the coordinate plane.

- **Provisional — G.6(D):** apply the Pythagorean Theorem and its converse to solve problems and to determine whether a triangle is a right triangle.
- **Provisional — G.2(B):** use the distance formula on the coordinate plane to find the length of a segment between two points.
- **Provisional — G.5(A):** use right-triangle relationships and length calculations to solve mathematical and real-world problems.

Provisional — pending educator verification against the current official TAC source.

(Standards are paraphrased here, not quoted. Please confirm codes and wording against your official source. This packet is **not** claimed to be "aligned to the TEKS" until reviewed.)

Setup Before class (5 min)

- Count and hand out one **student packet** per student.
- Confirm each student has a **pencil**. A **basic or scientific calculator is allowed but not required**; students who have one may use it (helpful for square roots).
- Write the date on the board and remind students to fill in the name line.
- Keep the **answer key** with you — it is a separate file and is **not** in the student packet.
- **Standard vs. block:** on a ~50-minute period, students do items 1–8, 10, and ACE (item 9 is optional). On a ~90-minute block, students also complete **item 9**, the multi-step composite-distance task.

Materials What's needed

- Printed student packet (5–7 pages) per student.
- Pencil per student; a basic four-function or scientific calculator is optional.
- This guide and the separate answer key (teacher only).

- No technology, no internet, no special supplies.

Timing Suggested pacing (standard ~50 min • block ~90 min)

Segment	Standard (~50 min)	Block (~90 min)	What students do
Start — Retrieval warm-up	6 min	8 min	Q1–Q2 (squaring & roots; horizontal/vertical distance)
Build — Study the math	8–12 min	12–15 min	Read reference box, worked example, Figure 1; Q3
Apply — Use what you know	20–26 min	28–34 min	Q4–Q8 (missing side, distance, ladder, error analysis, converse)
Block — Extra applied task	optional	12–15 min	Q9 (composite path vs. straight-line distance)
Explain — Justify (CER)	6–10 min	8–12 min	Q10 claim/evidence/reasoning (converse)
Close — ACE	5 min	5–7 min	Articulate / Connect / Extend
Continue — early finishers	optional	optional	Build Pythagorean triples; test 5-6-8

Script Read aloud to start

"Today's assignment is a paper math packet called *The Pythagorean Theorem and Distance*. You will work by yourself with a pencil. A basic or scientific calculator is allowed if you have one, but you do not need one. The most important rule: **show your work** — write down the squares you compute and the square root you take, not just a final number. When an answer is not a perfect square, give both the exact square root and a rounded decimal. Everything you need is printed in the packet, including a worked example, a labeled graph, and all the formulas. Start with the warm-up, then read the Build section before the Apply questions. If a question is tricky, skip it and come back. Put your name, class period, and date at the top now. You have about [50 / 90] minutes; raise your hand if you need help reading a question."

Support When students ask for help

- **Students show their work; you do not need to teach the method.** Point them back to the printed **reference box** and the **worked example** — the method is right there.
- **You may read any part aloud** to a student or the class.
- If a student is stuck, prompt with "What does the worked example do?" or "Which side is across from the right angle — that is the hypotenuse" rather than giving the answer.
- For a missing side, remind them: to find the hypotenuse you **add** the squares then square-root; to find a leg you **subtract** ($a^2 = c^2 - b^2$) then square-root.
- Remind students that the **distance formula is just $a^2 + b^2 = c^2$** — the horizontal change is one leg and the vertical change is the other.
- It is fine if a student cannot finish the optional early-finisher challenge.

Access Accommodations & language support

- **Read-aloud** of any question is allowed for all students.
- Point students to the **worked example**, the **formulas** in the reference box, and the **sentence stems** for Q10 and ACE.

- A **calculator is allowed** for any student who wants one — this removes the square-root arithmetic barrier so students can focus on the reasoning.
- Extended time is fine; the core can stop after Q10 if time runs short.
- Students may show work with numbers and short phrases if writing full sentences is a barrier, as long as the reasoning is clear.
- The packet is grayscale-safe — Figure 1 uses black lines, labeled vertices, a right-angle mark, and text, so it prints and reads clearly in black and white.

Tools Allowed & not allowed

Allowed: pencil, the printed packet, a basic or scientific calculator, quiet self-read-aloud. **Not needed / not allowed:** phones for messaging, internet, AI tools, or getting answers from another student.

Collect at the end: Collect every packet, whether finished or not, with names on them. Stack them for the teacher. Note on a sticky how far most students got and any questions that caused confusion.

Answer key: The **answer key is a separate file** (answerkey.html) and is for the teacher only. Please do not hand it to students.

Teacher follow-up (for the returning teacher)

The highest-value items to review next class are the **error analysis (Q7)** and the **justification (Q10)**, which reveal the "add the legs" error and whether students can use the converse instead of assuming a scalene triangle is right. Watch for **mislabeled hypotenuses** on the ladder item (Q6), **forgetting to square-root** (Q3, Q5), and dropping the **exact radical** when the answer is not a perfect square ($Q3 \rightarrow \sqrt{180}$, $Q9c \rightarrow \sqrt{116}$). The **distance** items (Q5, Q9) show who connects the distance formula back to $a^2 + b^2 = c^2$. The separate answer key lists worked steps, alternate strategies, misconceptions, and a 3-point rubric for Q10.

HS_GEOM_Pythagorean_01 — The Pythagorean Theorem & Distance on the Coordinate Plane Substitute Guide

The Pythagorean Theorem & Distance on the Coordinate Plane

Course: Geometry (Grades 9–12) **Subject:** Mathematics **Time:** ~50 min standard · ~90 min block **Work mode:** Independent · pencil + packet Math

Start Here

This is a **paper math packet** about the **Pythagorean Theorem** and using it to find **distance** on the coordinate plane. In a **right triangle**, the two shorter sides (the **legs**) and the longest side (the **hypotenuse**) are connected by the rule $a^2 + b^2 = c^2$. You will use it to find a missing side, to measure the **distance between two plotted points**, to solve a **real-world** problem (a ladder), to **fix an error**, and to test whether a triangle is a **right triangle** (the **converse**). Work by yourself with a pencil. **A calculator is not required, but a basic or scientific calculator is allowed** if you have one. **Show your work** — every answer should show the squares you computed and the square root you took, not just a final number. Give both an **exact** answer (such as $\sqrt{52}$) and a **rounded** answer (to the nearest tenth) when a number is not a perfect square. Everything you need (formulas, a worked example, a labeled graph) is printed right here. If a question is hard, skip it, keep going, and come back. Circle or box your final answers.

Start Warm-Up: Retrieval 6 min

Bring back squaring, square roots, and reading points off a grid.

1Square and square-root. Find 5^2 and 12^2 , then add them. Finally, find $\sqrt{169}$. Show each step. (Remember: 5^2 means 5×5 , not 5×2 .)

2Read a horizontal and a vertical distance. A point starts at **(2, 3)** and a second point is at **(2, 8)**. How far apart are they **up-and-down** (the vertical distance)? Then a third point is at **(6, 3)**: how far is it from **(2, 3)** **left-and-right** (the horizontal distance)? Show the subtraction you used.

Build Study the Math 8–12 min

What you need to know (with formulas)

A **right triangle** has one **90° angle** (a square corner). The two sides that form the right angle are the **legs** (call their lengths **a** and **b**). The side **across from** the right angle is the **hypotenuse** (length **c**). The hypotenuse is **always the longest side** — that is how you can spot it.

The Pythagorean Theorem. For any right triangle:

$$a^2 + b^2 = c^2$$

The two legs squared and added equal the hypotenuse squared. Use it two ways:

- **Missing hypotenuse (c):** add the squares of the legs, then take the square root: $c = \sqrt{a^2 + b^2}$.
- **Missing leg (a or b):** the hypotenuse is already known, so **subtract**: $a = \sqrt{c^2 - b^2}$. (Square the hypotenuse, subtract the known leg's square, then take the square root.)

The Distance Formula (an application). To find the distance between two points (x_1, y_1) and (x_2, y_2) on a grid, make a right triangle: the horizontal leg is the change in x and the vertical leg is the change in y . Then apply the theorem:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This is just $a^2 + b^2 = c^2$ where $a = (x_2 - x_1)$, $b = (y_2 - y_1)$, and $d = c$. Subtracting in the other order is fine because squaring makes it positive either way.

The Converse (a right-triangle test). The theorem also works *backward*. If the three side lengths of a triangle make $a^2 + b^2 = c^2$ true (with c the **longest** side), then the triangle **is a right triangle**. If $a^2 + b^2 \neq c^2$, it is **not** a right triangle. To use it: square all three sides, add the squares of the two shorter sides, and compare that sum to the square of the longest side.

Tip: when $a^2 + b^2$ is not a perfect square, the exact answer is a square root (like $\sqrt{52}$). Leave it as a root for the **exact** value, and also give a **rounded** decimal ($\sqrt{52} \approx 7.2$) to the nearest tenth.

Worked Example — a 6-8-10 right triangle on the grid

Look at Figure 1. The right angle is at **B(7, 1)**. One leg runs from **A(1, 1)** to **B(7, 1)**; the other leg runs from **B(7, 1)** to **C(7, 9)**. The hypotenuse is the slanted side from **A(1, 1)** to **C(7, 9)** — it is across from the right angle and is the longest side.

Find the legs by counting boxes: leg $a = AB = 7 - 1 = 6$ units (horizontal); leg $b = BC = 9 - 1 = 8$ units (vertical).

Find the hypotenuse c with the theorem:

$$a^2 + b^2 = c^2 \rightarrow 6^2 + 8^2 = c^2 \rightarrow 36 + 64 = c^2 \rightarrow 100 = c^2$$

$c = \sqrt{100} = 10$. So the hypotenuse AC is **10 units**. (Exact and rounded agree here because 100 is a perfect square.)

Same answer using the distance formula for A(1, 1) and C(7, 9): $d = \sqrt{(7 - 1)^2 + (9 - 1)^2} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$. ✓

Check the converse: the sides are 6, 8, 10, and 10 is longest. $6^2 + 8^2 = 36 + 64 = 100$, and $10^2 = 100$. Since $100 = 100$, the triangle **is** a right triangle. ✓

The three sides of triangle ABC (right angle at B).

Side	From → To	Type	Length (units)
AB	(1, 1) → (7, 1)	leg a (horizontal)	6
BC	(7, 1) → (7, 9)	leg b (vertical)	8
AC	(1, 1) → (7, 9)	hypotenuse c	10

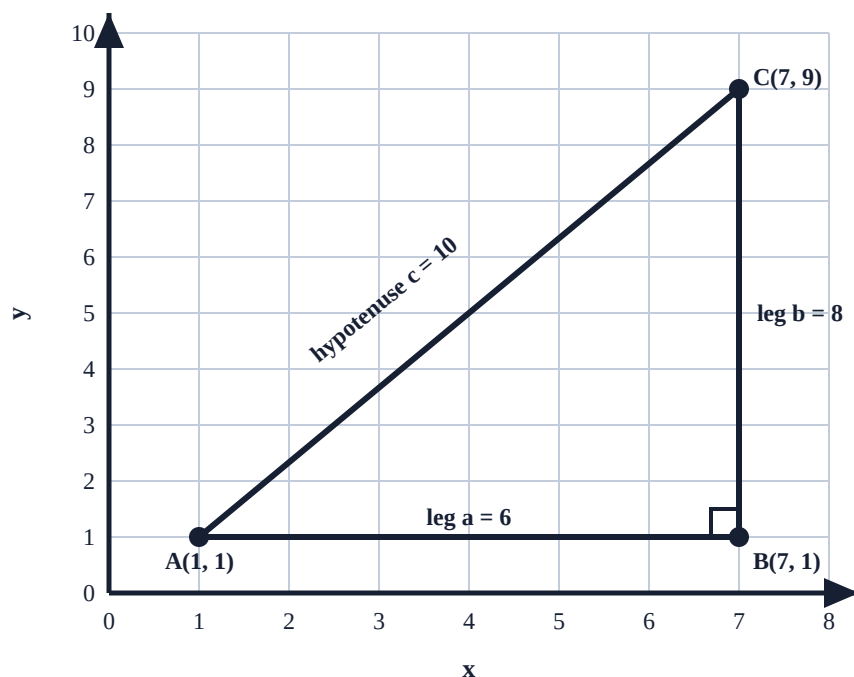


Figure 1. A 6-8-10 right triangle with vertices **A(1, 1)**, **B(7, 1)**, and **C(7, 9)**. The right angle (small square) is at B. Leg **a** = **AB** = **6**, leg **b** = **BC** = **8**, and the hypotenuse **c** = **AC** = **10**, because $6^2 + 8^2 = 36 + 64 = 100$ and $\sqrt{100} = 10$. The triangle and labels are drawn in black with text so the figure reads clearly in grayscale.

3Using Figure 1, suppose the vertical leg **BC** were **12** instead of 8 (with the horizontal leg **AB** still **6**). Use $a^2 + b^2 = c^2$ to find the new hypotenuse. Give the **exact** answer (a square root if needed) and the answer **rounded to the nearest tenth**. Show your work.

Apply Use What You Know 20–26 min

Show the squares and the square root in every item. A calculator is allowed. When the answer is not a perfect square, give both the exact root and the rounded decimal.

4Find a missing side.

4a. Missing hypotenuse. A right triangle has legs **a** = **9** and **b** = **12**. Find the hypotenuse **c**. Use $a^2 + b^2 = c^2$ and show every step. (This one comes out to a whole number.)

4b. Missing leg. A different right triangle has one leg **b** = **5** and hypotenuse **c** = **13**. Find the other leg **a**. (Careful: the hypotenuse is known, so you must **subtract**: $a^2 = c^2 - b^2$.) Show your steps.

5**Distance between two plotted points.** On a coordinate grid, point **P** is at **(1, 2)** and point **Q** is at **(7, 10)**. Find the **distance PQ**. Use the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$; show the horizontal change, the vertical

change, both squares, the sum, and the square root.

6Real-world application (a ladder). A **17-foot** ladder leans against a wall. Its base is **8 feet** from the bottom of the wall. **How high up the wall does the ladder reach?** The wall and ground make the right angle, the ground distance and the wall height are the **legs**, and the ladder is the **hypotenuse**. Draw a quick right triangle, label it, and solve. Show your work.

7Error analysis (added instead of using the theorem). A student found the hypotenuse of a right triangle with legs **$a = 6$** and **$b = 8$** and wrote the work shown. **Find the mistake, explain it, and give the correct hypotenuse.**

Student's work (legs 6 and 8):

$c = a + b = 6 + 8 = 14$. So the hypotenuse is 14.

What is wrong, and what is the correct hypotenuse? Fix it in the box, showing the correct use of $a^2 + b^2 = c^2$.

8Use the converse to test a triangle. A triangle has side lengths **10, 24, and 26**. **Is it a right triangle?** Identify the **longest** side, square all three sides, and check whether $a^2 + b^2 = c^2$. State your conclusion clearly (yes or no) and why.

Block Extra Applied Task block only • ~12–15 min

If your class is on a ~90-minute block schedule, complete Item 9. (On a standard ~50-minute period, this item is optional.)

9Multi-step composite distance. A hiker walks a path with three stops plotted on a grid (each unit = 1 kilometer): **R(0, 0)**, then **S(3, 4)**, then **T(10, 4)**. She walks a straight segment from R to S, then a straight segment from S to T.

9a. Find the length of segment **RS** using the distance formula. Show your work.

9b. Find the length of segment **ST** using the distance formula. Show your work.

9c. Find the **total distance walked** ($RS + ST$). Then find the **straight-line distance** from R directly to T (the distance formula for R to T). Which is longer, the path or the straight line? Explain in one sentence why that makes sense.

Explain Justify Your Strategy 6–10 min

Question 10. A triangle has side lengths **7, 9, and 12**. A classmate says, "It must be a right triangle because it has three different sides." **Is the triangle a right triangle? Decide, and justify your answer** using the converse of the Pythagorean Theorem. Write a **Claim** (your yes/no answer), **Evidence** (the squaring and adding you computed), and **Reasoning** (why that computation proves your claim, and why the classmate's reason is not enough).

Sentence stems you may use: "The triangle is / is not a right triangle because..." · "The longest side is ____, so $c = ___.$ " · "I found $a^2 + b^2 = ___$ and $c^2 = ___.$ " · "Since $a^2 + b^2 ___ c^2$, the converse tells me..." · "Having three different sides does not prove a right angle because..."

Claim (your yes/no answer)

Evidence (the numbers you computed)

Reasoning (why the computation proves it)

Close ACE Wrap-Up 5 min

Articulate

In your own words, explain how you can tell which side of a right triangle is the **hypotenuse**, and why you must **square-root** at the end instead of stopping at c^2 .

Connect

Point to one item in this packet where the **distance formula** was really just $a^2 + b^2 = c^2$ in disguise. Give its number and name the legs.

Extend

Give a **new** real-life situation (not the ladder) where you could use the Pythagorean Theorem to find a distance you cannot measure directly.

Continue Early Finisher optional

If you finish early (a challenge — no new materials needed):

Find a Pythagorean triple of your own. A **Pythagorean triple** is three whole numbers a, b, c with $a^2 + b^2 = c^2$ (like 3-4-5 or 6-8-10). On the back of this page: (1) start with the triple **3-4-5** and **multiply all three numbers by the same whole number** (try 2, then 3) to make two **new** triples; (2) **prove** each new set really works by showing $a^2 + b^2 = c^2$; and (3) explain in one sentence **why** multiplying a triple by the same number always gives another right triangle. Then use the **converse** to show that **5-6-8** is **not** a right triangle.

Turn in: Hand in this whole packet with your name, class period, and date filled in. Make sure items 1–8 and 10 and the ACE box are answered **with your work shown** (item 9 too if you are on a block schedule), including

both exact and rounded answers where needed. The early-finisher challenge is optional but turn it in too if you did it.

HS_GEOM_Pythagorean_01 — The Pythagorean Theorem & Distance on the Coordinate Plane Student Packet



TEACHER ANSWER KEY FOLLOWS

Do not copy the pages after this divider for students.

Answer Key: The Pythagorean Theorem & Distance

Course: Geometry (Grades 9–12) **Subject:** Mathematics **For teacher use only** Math

How to use this key

Answers are grouped by section and question number, with full worked steps. Accept any correct method that reaches the right value — a student may use $a^2 + b^2 = c^2$ directly or the distance formula, since the distance formula **is** the theorem. Where an answer is not a perfect square, both the **exact** root and a **rounded** decimal (nearest tenth) are given; accept either as long as the exact form is shown. Watch-for notes flag common misconceptions. A basic or scientific calculator is allowed, so grade the **reasoning and steps**, not just the arithmetic. Estimated grading time: ~6–8 min per packet.

Start Warm-Up: Retrieval ~1 min

1 Square, add, and square-root.

$5^2 = 25$ and $12^2 = 144$; $25 + 144 = \mathbf{169}$; $\sqrt{169} = \mathbf{13}$. Full credit for showing 5×5 and 12×12 . *Watch-for:* a student who writes $5^2 = 10$ (multiplying by 2 instead of squaring) or $12^2 = 24$. Squaring means the number times **itself**.

2 Vertical and horizontal distances.

Vertical from (2, 3) to (2, 8): $8 - 3 = 5$ units (x stays the same). Horizontal from (2, 3) to (6, 3): $6 - 2 = 4$ units (y stays the same). *Watch-for:* subtracting the wrong pair of coordinates; on a vertical segment the y-values change, on a horizontal segment the x-values change. This sets up the legs used in the distance formula.

Build Study the Math ~1 min

3 New hypotenuse with legs $a = 6$, $b = 12$.

$6^2 + 12^2 = c^2 \rightarrow 36 + 144 = c^2 \rightarrow 180 = c^2$. $c = \sqrt{180} = \sqrt{(36 \times 5)} = \mathbf{6\sqrt{5}}$ (exact) $\approx \mathbf{13.4}$ units (rounded to the nearest tenth). *Accept* $\sqrt{180}$ left unsimplified as long as the decimal ≈ 13.4 is given. *Watch-for:* stopping at $c^2 = 180$ and forgetting the square root, or writing $c = 18$ by mis-simplifying.

Apply Use What You Know ~3–4 min

4 Find a missing side.

4a (missing hypotenuse): $a^2 + b^2 = c^2 \rightarrow 9^2 + 12^2 = c^2 \rightarrow 81 + 144 = 225 \rightarrow c = \sqrt{225} = \mathbf{15}$ (a whole number; exact and rounded agree).

4b (missing leg): the hypotenuse is known, so subtract: $a^2 = c^2 - b^2 = 13^2 - 5^2 = 169 - 25 = 144 \rightarrow a = \sqrt{144} = \mathbf{12}$. *Watch-for:* a student who **adds** ($13^2 + 5^2 = 194$) instead of subtracting — remind them the hypotenuse is already the largest side, so you take away the known leg's square. *Check:* 5, 12, 13 is a Pythagorean triple.

5 Distance PQ for P(1, 2) and Q(7, 10).

Horizontal change $x_2 - x_1 = 7 - 1 = 6$; vertical change $y_2 - y_1 = 10 - 2 = 8$. $d = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$ units (exact and rounded agree). *Alternate*: subtracting in the other order, $\sqrt{(1 - 7)^2 + (2 - 10)^2} = \sqrt{(-6)^2 + (-8)^2} = \sqrt{100} = 10$ — squaring removes the sign. *Watch-for*: a student who forgets the square root and answers 100.

6 Ladder: 17-ft ladder, base 8 ft from the wall.

The ladder is the **hypotenuse** ($c = 17$); the ground distance is a leg ($b = 8$); the wall height is the other leg (a). $a^2 = c^2 - b^2 = 17^2 - 8^2 = 289 - 64 = 225 \rightarrow a = \sqrt{225} = 15$ feet up the wall (exact and rounded agree; 8-15-17 is a triple). *Watch-for*: treating 17 as a leg and adding ($17^2 + 8^2 = 353 \rightarrow \approx 18.8$), which mislabels the hypotenuse. The ladder, leaning across from the right angle, must be the longest side.

7 Error analysis: added the legs instead of using the theorem.

The mistake: the student **added the legs** ($6 + 8 = 14$) instead of squaring them, adding the squares, and taking the square root. The Pythagorean Theorem uses $a^2 + b^2 = c^2$, not $a + b = c$.

Correct work: $c^2 = 6^2 + 8^2 = 36 + 64 = 100 \rightarrow c = \sqrt{100} = 10$ (not 14).

Full credit requires (1) naming that they added the legs instead of squaring/adding/ rooting, and (2) the correct hypotenuse of 10. *Sense-check to offer*: the hypotenuse must be longer than each leg but shorter than the two legs added together — 10 sits between 8 and 14, while the student's 14 equals the sum, which only happens for a flat (degenerate) triangle.

8 Converse test on sides 10, 24, 26.

Longest side $c = 26$. Two shorter sides: 10 and 24. $a^2 + b^2 = 10^2 + 24^2 = 100 + 576 = 676$. $c^2 = 26^2 = 676$. Since $676 = 676$, $a^2 + b^2 = c^2$, so the triangle **IS a right triangle** (10-24-26 is a Pythagorean triple, twice 5-12-13).

Watch-for: choosing the wrong side as c ; the longest side must be the hypotenuse in the test.

Block Extra Applied Task block only • ~2 min

9 Composite distance: R(0, 0), S(3, 4), T(10, 4) (km).

9a. RS: $d = \sqrt{(3 - 0)^2 + (4 - 0)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$ km.

9b. ST: $d = \sqrt{(10 - 3)^2 + (4 - 4)^2} = \sqrt{49 + 0} = \sqrt{49} = 7$ km (a horizontal segment, so the vertical change is 0).

9c. Total path RS + ST = 5 + 7 = **12 km**. **Straight line RT**: $d = \sqrt{(10 - 0)^2 + (4 - 0)^2} = \sqrt{100 + 16} = \sqrt{116} = 2\sqrt{29}$ (exact) ≈ 10.8 km (rounded). The **path (12 km) is longer** than the straight line (≈ 10.8 km). *Reasoning students should give*: a straight line is the shortest distance between two points, so bending at S makes the trip longer than going directly from R to T. *Watch-for*: adding RS + ST and thinking it should equal RT (that only happens when the three points are collinear, which they are not here).

Explain Justify Your Strategy (Q10) ~1–2 min

10 Is a 7-9-12 triangle a right triangle?

Model answer. Claim: No — the triangle is **not** a right triangle. **Evidence**: the longest side is 12, so $c = 12$. The two shorter sides give $a^2 + b^2 = 7^2 + 9^2 = 49 + 81 = 130$, while $c^2 = 12^2 = 144$. Since $130 \neq 144$, the sides do not satisfy $a^2 + b^2 = c^2$. **Reasoning**: the converse of the Pythagorean Theorem says a triangle is right **only when** the squares of the two shorter sides add up to the square of the longest side. Here the sum (130) is **less** than 144, so the triangle is not right (in fact it is obtuse, since $a^2 + b^2 < c^2$). Having three different side lengths only makes it **scalene**; scalene says nothing about the angles, so the classmate's reason does not prove a right angle.

Note: Full credit requires the correct "no," the computation 130 vs. 144, and a statement that the converse (not "three different sides") is what decides it. Students are not required to name it obtuse, but it is a strong extension.

Justification scoring rubric (3 points)

Score	Claim	Evidence	Reasoning
3	States clearly it is not a right triangle.	Correctly computes $a^2 + b^2 = 130$ and $c^2 = 144$ ($c = \text{longest} = 12$) and notes $130 \neq 144$.	Cites the converse and explains that "three different sides" (scalene) does not prove a right angle.
2	Correct "not right" answer.	One square or the comparison shown, with a minor slip.	Reasoning present but does not address the classmate's flawed reason or the converse by name.
1	An answer with weak or no support.	Evidence largely missing or incorrect (e.g., wrong side chosen as c).	Little or flawed reasoning.
0	No/incorrect claim.	No evidence.	No reasoning.

Close ACE ~1 min

ACE Articulate / Connect / Extend.

Articulate: Accept any correct explanation: "The **hypotenuse** is the side **across from the right angle** and is the **longest** side. You must take the **square root** at the end because $a^2 + b^2 = c^2$ gives you c^2 (the square of the side), not c itself; $\sqrt{c^2} = c$ undoes the squaring to get the actual length."

Connect: Q5 (distance PQ) — legs are the horizontal change 6 and vertical change 8; also Q9a/9b/9c. Any of these earns credit if the legs are named. Q6's ladder and Q3 use the theorem directly rather than via coordinates.

Extend: Any genuine "can't-measure-directly" distance — e.g., the diagonal of a rectangular room or TV screen, the straight-line distance across a park cut by two streets meeting at a right angle, a ramp's length from its run and rise, a guy-wire on a pole, or the diagonal of a baseball diamond. Must involve a right triangle where two sides are known and the third is found.

Challenge Early Finisher (optional)

EF Build Pythagorean triples from 3-4-5; test 5-6-8.

(1) **New triples by scaling 3-4-5:** $\times 2 \rightarrow 6-8-10$; $\times 3 \rightarrow 9-12-15$.

(2) **Proofs:** $6^2 + 8^2 = 36 + 64 = 100 = 10^2$ ✓; $9^2 + 12^2 = 81 + 144 = 225 = 15^2$ ✓.

(3) **Why scaling works:** multiplying every side by k multiplies both sides of $a^2 + b^2 = c^2$ by k^2 ($(ka)^2 + (kb)^2 = k^2(a^2 + b^2) = k^2c^2 = (kc)^2$), so the equation still holds — the new triangle is **similar** to the original and keeps its right angle.

Converse on 5-6-8: longest side 8, so $5^2 + 6^2 = 25 + 36 = 61$, but $8^2 = 64$. Since $61 \neq 64$, **5-6-8 is NOT a right triangle** (it is close, but not exact). *Full credit* requires two correct scaled triples with proofs, a reason scaling preserves the right angle, and the correct "not right" conclusion for 5-6-8.

Watch Common misconceptions

- **Adding the legs instead of using the theorem.** The Q7 error: writing $c = a + b$ ($6 + 8 = 14$) instead of $c = \sqrt{a^2 + b^2} = 10$. Anchor: you must **square** the legs, **add** the squares, and then **take the square root**. The sum of the legs is always *more* than the hypotenuse.
- **Mislabeling the hypotenuse.** The Q6 trap: treating the longest given length (the ladder, 17) as a leg and adding. The hypotenuse is **across from the right angle** and is the **longest** side; when it is known you **subtract** ($a^2 = c^2 - b^2$). Reinforce on Q4b, Q6, and Q8.
- **Forgetting to square-root.** Students stop at $c^2 = 180$ (Q3) or $d^2 = 100$ (Q5) and report 180 or 100. The theorem gives the **square** of the side; the final step is $\sqrt{}$ to get the length itself.
- **Squaring vs. doubling.** On Q1 and throughout, some write $5^2 = 10$ or $12^2 = 24$. Squaring means the number times **itself** ($5 \times 5 = 25$), not times 2.

- **Dropping the exact root.** When $a^2 + b^2$ is not a perfect square (Q3 $\rightarrow \sqrt{180}$, Q9c $\rightarrow \sqrt{116}$), require the **exact** radical *and* the rounded decimal; a bare decimal loses the exact value, and a bare "180" is an unfinished answer.
- **Choosing the wrong side as c in the converse.** On Q8 and Q10, the longest side must be squared alone on one side of the test. Testing $7^2 + 12^2$ vs. 9^2 would give a false result — always add the two **shorter** sides and compare to the **longest**.
- **Assuming scalene means right (Q10).** Three different side lengths make a triangle scalene, which says nothing about its angles. Only the converse ($a^2 + b^2 = c^2$) decides whether it is right.

Teacher follow-up based on likely errors

If many students miss Q7, do a quick 5-minute practice: for legs 3 and 4, contrast $3 + 4 = 7$ with $\sqrt{3^2 + 4^2} = 5$, and have students explain why the sum of the legs cannot be the hypotenuse. If Q6 shows mislabeled hypotenuses, sketch a ladder and label the longest side as the hypotenuse before setting up. The Q8 and Q10 converse items reveal who can pick the longest side as c and who confuses "scalene" with "right." The Q5 and Q9 items show who connects the distance formula back to $a^2 + b^2 = c^2$. The exact-vs-rounded answers on Q3 and Q9c show who finishes with both a radical and a decimal — all good warm-up discussions next class.

HS_GEOM_Pythagorean_01 — The Pythagorean Theorem & Distance on the Coordinate Plane Teacher Answer Key