

Answer Key: The Pythagorean Theorem & Distance

Course: Geometry (Grades 9–12) **Subject:** Mathematics **For teacher use only** Math

How to use this key

Answers are grouped by section and question number, with full worked steps. Accept any correct method that reaches the right value — a student may use $a^2 + b^2 = c^2$ directly or the distance formula, since the distance formula **is** the theorem. Where an answer is not a perfect square, both the **exact** root and a **rounded** decimal (nearest tenth) are given; accept either as long as the exact form is shown. Watch-for notes flag common misconceptions. A basic or scientific calculator is allowed, so grade the **reasoning and steps**, not just the arithmetic. Estimated grading time: ~6–8 min per packet.

Start Warm-Up: Retrieval ~1 min

1 Square, add, and square-root.

$5^2 = 25$ and $12^2 = 144$; $25 + 144 = \mathbf{169}$; $\sqrt{169} = \mathbf{13}$. Full credit for showing 5×5 and 12×12 . *Watch-for:* a student who writes $5^2 = 10$ (multiplying by 2 instead of squaring) or $12^2 = 24$. Squaring means the number times **itself**.

2 Vertical and horizontal distances.

Vertical from (2, 3) to (2, 8): $8 - 3 = 5$ units (x stays the same). Horizontal from (2, 3) to (6, 3): $6 - 2 = 4$ units (y stays the same). *Watch-for:* subtracting the wrong pair of coordinates; on a vertical segment the y-values change, on a horizontal segment the x-values change. This sets up the legs used in the distance formula.

Build Study the Math ~1 min

3 New hypotenuse with legs $a = 6$, $b = 12$.

$6^2 + 12^2 = c^2 \rightarrow 36 + 144 = c^2 \rightarrow 180 = c^2$. $c = \sqrt{180} = \sqrt{(36 \times 5)} = \mathbf{6\sqrt{5}}$ (exact) $\approx \mathbf{13.4}$ units (rounded to the nearest tenth). *Accept* $\sqrt{180}$ left unsimplified as long as the decimal ≈ 13.4 is given. *Watch-for:* stopping at $c^2 = 180$ and forgetting the square root, or writing $c = 18$ by mis-simplifying.

Apply Use What You Know ~3–4 min

4 Find a missing side.

4a (missing hypotenuse): $a^2 + b^2 = c^2 \rightarrow 9^2 + 12^2 = c^2 \rightarrow 81 + 144 = 225 \rightarrow c = \sqrt{225} = \mathbf{15}$ (a whole number; exact and rounded agree).

4b (missing leg): the hypotenuse is known, so subtract: $a^2 = c^2 - b^2 = 13^2 - 5^2 = 169 - 25 = 144 \rightarrow a = \sqrt{144} = \mathbf{12}$. *Watch-for:* a student who **adds** ($13^2 + 5^2 = 194$) instead of subtracting — remind them the hypotenuse is already the largest side, so you take away the known leg's square. *Check:* 5, 12, 13 is a Pythagorean triple.

5 Distance PQ for P(1, 2) and Q(7, 10).

Horizontal change $x_2 - x_1 = 7 - 1 = 6$; vertical change $y_2 - y_1 = 10 - 2 = 8$. $d = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$ units (exact and rounded agree). *Alternate*: subtracting in the other order, $\sqrt{(1 - 7)^2 + (2 - 10)^2} = \sqrt{(-6)^2 + (-8)^2} = \sqrt{100} = 10$ — squaring removes the sign. *Watch-for*: a student who forgets the square root and answers 100.

6 Ladder: 17-ft ladder, base 8 ft from the wall.

The ladder is the **hypotenuse** ($c = 17$); the ground distance is a leg ($b = 8$); the wall height is the other leg (a). $a^2 = c^2 - b^2 = 17^2 - 8^2 = 289 - 64 = 225 \rightarrow a = \sqrt{225} = 15$ feet up the wall (exact and rounded agree; 8-15-17 is a triple). *Watch-for*: treating 17 as a leg and adding ($17^2 + 8^2 = 353 \rightarrow \approx 18.8$), which mislabels the hypotenuse. The ladder, leaning across from the right angle, must be the longest side.

7 Error analysis: added the legs instead of using the theorem.

The mistake: the student **added the legs** ($6 + 8 = 14$) instead of squaring them, adding the squares, and taking the square root. The Pythagorean Theorem uses $a^2 + b^2 = c^2$, not $a + b = c$.

Correct work: $c^2 = 6^2 + 8^2 = 36 + 64 = 100 \rightarrow c = \sqrt{100} = 10$ (not 14).

Full credit requires (1) naming that they added the legs instead of squaring/adding/ rooting, and (2) the correct hypotenuse of 10. *Sense-check to offer*: the hypotenuse must be longer than each leg but shorter than the two legs added together — 10 sits between 8 and 14, while the student's 14 equals the sum, which only happens for a flat (degenerate) triangle.

8 Converse test on sides 10, 24, 26.

Longest side $c = 26$. Two shorter sides: 10 and 24. $a^2 + b^2 = 10^2 + 24^2 = 100 + 576 = 676$. $c^2 = 26^2 = 676$. Since $676 = 676$, $a^2 + b^2 = c^2$, so the triangle **IS a right triangle** (10-24-26 is a Pythagorean triple, twice 5-12-13).

Watch-for: choosing the wrong side as c ; the longest side must be the hypotenuse in the test.

Block Extra Applied Task block only • ~2 min

9 Composite distance: R(0, 0), S(3, 4), T(10, 4) (km).

9a. RS: $d = \sqrt{(3 - 0)^2 + (4 - 0)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$ km.

9b. ST: $d = \sqrt{(10 - 3)^2 + (4 - 4)^2} = \sqrt{49 + 0} = \sqrt{49} = 7$ km (a horizontal segment, so the vertical change is 0).

9c. Total path RS + ST = 5 + 7 = 12 km. Straight line RT: $d = \sqrt{(10 - 0)^2 + (4 - 0)^2} = \sqrt{100 + 16} = \sqrt{116} = 2\sqrt{29}$ (exact) ≈ 10.8 km (rounded). The **path (12 km) is longer** than the straight line (≈ 10.8 km). *Reasoning students should give*: a straight line is the shortest distance between two points, so bending at S makes the trip longer than going directly from R to T. *Watch-for*: adding RS + ST and thinking it should equal RT (that only happens when the three points are collinear, which they are not here).

Explain Justify Your Strategy (Q10) ~1–2 min

10 Is a 7-9-12 triangle a right triangle?

Model answer. Claim: No — the triangle is **not** a right triangle. **Evidence**: the longest side is 12, so $c = 12$. The two shorter sides give $a^2 + b^2 = 7^2 + 9^2 = 49 + 81 = 130$, while $c^2 = 12^2 = 144$. Since $130 \neq 144$, the sides do not satisfy $a^2 + b^2 = c^2$. **Reasoning**: the converse of the Pythagorean Theorem says a triangle is right **only when** the squares of the two shorter sides add up to the square of the longest side. Here the sum (130) is **less** than 144, so the triangle is not right (in fact it is obtuse, since $a^2 + b^2 < c^2$). Having three different side lengths only makes it **scalene**; scalene says nothing about the angles, so the classmate's reason does not prove a right angle.

Note: Full credit requires the correct "no," the computation 130 vs. 144, and a statement that the converse (not "three different sides") is what decides it. Students are not required to name it obtuse, but it is a strong extension.

Justification scoring rubric (3 points)

Score	Claim	Evidence	Reasoning
3	States clearly it is not a right triangle.	Correctly computes $a^2 + b^2 = 130$ and $c^2 = 144$ ($c = \text{longest} = 12$) and notes $130 \neq 144$.	Cites the converse and explains that "three different sides" (scalene) does not prove a right angle.
2	Correct "not right" answer.	One square or the comparison shown, with a minor slip.	Reasoning present but does not address the classmate's flawed reason or the converse by name.
1	An answer with weak or no support.	Evidence largely missing or incorrect (e.g., wrong side chosen as c).	Little or flawed reasoning.
0	No/incorrect claim.	No evidence.	No reasoning.

Close ACE ~1 min

ACE Articulate / Connect / Extend.

Articulate: Accept any correct explanation: "The **hypotenuse** is the side **across from the right angle** and is the **longest** side. You must take the **square root** at the end because $a^2 + b^2 = c^2$ gives you c^2 (the square of the side), not c itself; $\sqrt{c^2} = c$ undoes the squaring to get the actual length."

Connect: Q5 (distance PQ) — legs are the horizontal change 6 and vertical change 8; also Q9a/9b/9c. Any of these earns credit if the legs are named. Q6's ladder and Q3 use the theorem directly rather than via coordinates.

Extend: Any genuine "can't-measure-directly" distance — e.g., the diagonal of a rectangular room or TV screen, the straight-line distance across a park cut by two streets meeting at a right angle, a ramp's length from its run and rise, a guy-wire on a pole, or the diagonal of a baseball diamond. Must involve a right triangle where two sides are known and the third is found.

Challenge Early Finisher (optional)

EF Build Pythagorean triples from 3-4-5; test 5-6-8.

(1) **New triples by scaling 3-4-5:** $\times 2 \rightarrow 6-8-10$; $\times 3 \rightarrow 9-12-15$.

(2) **Proofs:** $6^2 + 8^2 = 36 + 64 = 100 = 10^2$ ✓; $9^2 + 12^2 = 81 + 144 = 225 = 15^2$ ✓.

(3) **Why scaling works:** multiplying every side by k multiplies both sides of $a^2 + b^2 = c^2$ by k^2 ($(ka)^2 + (kb)^2 = k^2(a^2 + b^2) = k^2c^2 = (kc)^2$), so the equation still holds — the new triangle is **similar** to the original and keeps its right angle.

Converse on 5-6-8: longest side 8, so $5^2 + 6^2 = 25 + 36 = 61$, but $8^2 = 64$. Since $61 \neq 64$, **5-6-8 is NOT a right triangle** (it is close, but not exact). *Full credit* requires two correct scaled triples with proofs, a reason scaling preserves the right angle, and the correct "not right" conclusion for 5-6-8.

Watch Common misconceptions

- **Adding the legs instead of using the theorem.** The Q7 error: writing $c = a + b$ ($6 + 8 = 14$) instead of $c = \sqrt{a^2 + b^2} = 10$. Anchor: you must **square** the legs, **add** the squares, and then **take the square root**. The sum of the legs is always *more* than the hypotenuse.
- **Mislabeling the hypotenuse.** The Q6 trap: treating the longest given length (the ladder, 17) as a leg and adding. The hypotenuse is **across from the right angle** and is the **longest** side; when it is known you **subtract** ($a^2 = c^2 - b^2$). Reinforce on Q4b, Q6, and Q8.
- **Forgetting to square-root.** Students stop at $c^2 = 180$ (Q3) or $d^2 = 100$ (Q5) and report 180 or 100. The theorem gives the **square** of the side; the final step is $\sqrt{}$ to get the length itself.
- **Squaring vs. doubling.** On Q1 and throughout, some write $5^2 = 10$ or $12^2 = 24$. Squaring means the number times **itself** ($5 \times 5 = 25$), not times 2.

- **Dropping the exact root.** When $a^2 + b^2$ is not a perfect square ($Q3 \rightarrow \sqrt{180}$, $Q9c \rightarrow \sqrt{116}$), require the **exact** radical *and* the rounded decimal; a bare decimal loses the exact value, and a bare "180" is an unfinished answer.
- **Choosing the wrong side as c in the converse.** On Q8 and Q10, the longest side must be squared alone on one side of the test. Testing $7^2 + 12^2$ vs. 9^2 would give a false result — always add the two **shorter** sides and compare to the **longest**.
- **Assuming scalene means right (Q10).** Three different side lengths make a triangle scalene, which says nothing about its angles. Only the converse ($a^2 + b^2 = c^2$) decides whether it is right.

Teacher follow-up based on likely errors

If many students miss Q7, do a quick 5-minute practice: for legs 3 and 4, contrast $3 + 4 = 7$ with $\sqrt{3^2 + 4^2} = 5$, and have students explain why the sum of the legs cannot be the hypotenuse. If Q6 shows mislabeled hypotenuses, sketch a ladder and label the longest side as the hypotenuse before setting up. The Q8 and Q10 converse items reveal who can pick the longest side as c and who confuses "scalene" with "right." The Q5 and Q9 items show who connects the distance formula back to $a^2 + b^2 = c^2$. The exact-vs-rounded answers on Q3 and Q9c show who finishes with both a radical and a decimal — all good warm-up discussions next class.

HS_GEOM_Pythagorean_01 — The Pythagorean Theorem & Distance on the Coordinate Plane Teacher Answer Key