

Substitute Guide: Quadratic Functions — Graphs, Zeros, and the Vertex

Course: Algebra II (Grades 9–12) **Subject:** Mathematics **Time:** ~50 min standard · ~90 min block Math

Learning goal (plain language)

Students practice **quadratic functions** — reading a quadratic in **standard form** $y = ax^2 + bx + c$, finding its **vertex** (turning point) and **axis of symmetry**, finding its **zeros (roots)** by **factoring**, and **interpreting the vertex** as a maximum or minimum in a real story (a projectile's greatest height, a largest area). **You do not need a math background to run this.** Every formula and a worked example are printed in the student packet. Students **show their work**; you do not need to teach the method — the packet does the teaching. Your job is to hand it out, read the start script, keep the room on task, and collect the packets.

Standards alignment

Framework: 19 TAC §111.40 (Algebra II). Knowledge & skills for quadratic functions — graphing quadratics and identifying key attributes (vertex, axis of symmetry, zeros, y-intercept, maximum/minimum), solving quadratic equations by factoring, and interpreting these attributes in real-world contexts.

- **Provisional — graphing key attributes:** graph quadratic functions from standard form and identify the vertex, axis of symmetry, zeros/x-intercepts, y-intercept, and whether the vertex is a maximum or minimum.
- **Provisional — zeros by factoring:** solve quadratic equations of the form $ax^2 + bx + c = 0$ by factoring, using the Zero Product Property to find the roots.
- **Provisional — vertex in context:** interpret the vertex of a quadratic model as a maximum or minimum value (e.g., greatest height, maximum area) in a real-world situation.
- **Provisional — vertex form (block):** identify the vertex and axis of symmetry directly from vertex form $y = a(x - h)^2 + k$.

Provisional — pending educator verification against the current official TAC source.

(Standards are paraphrased here, not quoted. If unsure of exact section numbers, treat as 19 TAC Chapter 111 (Algebra II). Please confirm codes and wording against your official source. This packet is **not** claimed to be "aligned to the TEKS" until reviewed.)

Setup Before class (5 min)

- Count and hand out one **student packet** per student.
- Confirm each student has a **pencil**. A **graphing or scientific calculator is allowed but not required**; students who have one may use it.
- Write the date on the board and remind students to fill in the name line.
- Keep the **answer key** with you — it is a separate file and is **not** in the student packet.
- **Standard vs. block:** on a ~50-minute period, students do items 1–7, 9, and ACE (item 8 is optional). On a ~90-minute block, students also complete **item 8** — vertex form and the maximum-area modeling task.

Materials What's needed

- Printed student packet (5–7 pages) per student.
- Pencil per student; a graphing or scientific calculator is optional.
- This guide and the separate answer key (teacher only).
- No technology, no internet, no special supplies.

Timing Suggested pacing (standard ~50 min • block ~90 min)

Segment	Standard (~50 min)	Block (~90 min)	What students do
Start — Retrieval warm-up	6 min	8 min	Q1–Q2 (evaluate a quadratic; factor + Zero Product)
Build — Study the math	8–12 min	12–15 min	Read reference box, worked example, parabola graph; Q3
Apply — Use what you know	20–26 min	28–34 min	Q4–Q7 (read a graph, factor for roots, projectile context, error analysis)
Block — Extra applied task	optional	12–15 min	Q8 (vertex form; maximum-area modeling)
Explain — Justify (CER)	6–10 min	8–12 min	Q9 claim/evidence/reasoning
Close — ACE	5 min	5–7 min	Articulate / Connect / Extend
Continue — early finishers	optional	optional	Build your own quadratic from two roots

Script Read aloud to start

"Today's assignment is a paper math packet called *Quadratic Functions: Graphs, Zeros, and the Vertex*. You will work by yourself with a pencil. A graphing or scientific calculator is allowed if you have one, but you do not need one. The most important rule: **show your work** — write down the numbers and steps you use, not just a final answer. Everything you need is printed in the packet, including a worked example, a labeled parabola graph, and all the formulas. Start with the warm-up, then read the Build section before the Apply questions. If a question is tricky, skip it and come back. Put your name, class period, and date at the top now. You have about [50 / 90] minutes; raise your hand if you need help reading a question."

Support When students ask for help

- **Students show their work; you do not need to teach the method.** Point them back to the printed **reference box** and the **worked example** — the method is right there.
- **You may read any part aloud** to a student or the class.
- If a student is stuck, prompt with "What does the worked example do?" or "In $y = ax^2 + bx + c$, which sign of a tells you whether the vertex is a max or a min?" rather than giving the answer.
- For the vertex, remind them the packet shows $x = -b \div (2a)$, and that the axis of symmetry is that same vertical line.
- For zeros, remind them to **set the quadratic equal to 0, factor, then set each factor to 0** (Zero Product Property).
- Remind students that a **root is where $y = 0$** (on the x -axis) while the **vertex is the turning point**, halfway between the roots — they are not the same.
- It is fine if a student cannot finish the optional early-finisher challenge.

Access Accommodations & language support

- **Read-aloud** of any question is allowed for all students.
- Point students to the **worked example**, the **formulas** in the reference box, and the **sentence stems** for Q9 and ACE.
- A **calculator is allowed** for any student who wants one — this removes the arithmetic barrier so students can focus on the reasoning about vertex, axis, and roots.
- Extended time is fine; the core can stop after Q9 if time runs short.
- Students may show work with numbers and short phrases if writing full sentences is a barrier, as long as the reasoning is clear.
- The packet is grayscale-safe — the parabola uses black lines, labeled points, a dashed axis of symmetry, and text, so it prints and reads clearly in black and white.

Tools Allowed & not allowed

Allowed: pencil, the printed packet, a graphing or scientific calculator, quiet self-read-aloud. **Not needed / not allowed:** phones for messaging, internet, AI tools, or getting answers from another student.

Collect at the end: Collect every packet, whether finished or not, with names on them. Stack them for the teacher. Note on a sticky how far most students got and any questions that caused confusion.

Answer key: The **answer key is a separate file** (answerkey.html) and is for the teacher only. Please do not hand it to students.

Teacher follow-up (for the returning teacher)

The highest-value items to review next class are the **error analysis (Q7)** and the **justification (Q9)**, which reveal the **sign error in factoring** (opposite-sign factors when c is negative) and the **root-vs-vertex** confusion (the axis is the midpoint of the roots, not a root). Watch for students who assume the **vertex is always a minimum** (Q6 has $a < 0$, a maximum), who **stop after finding the vertex x** without substituting back for the height/area (Q6, Q8b), and who mishandle the sign in $x = -b \div (2a)$ or in vertex form (Q8a). The **read-a-graph** item (Q4) shows who can pull zeros, vertex, axis, and y-intercept from a table. The separate answer key lists worked steps, alternate strategies, misconceptions, and a 3-point rubric for Q9.

HS_ALG2_Quadratics_01 — Quadratic Functions: Graphs, Zeros, and the Vertex Substitute Guide

Quadratic Functions: Graphs, Zeros, and the Vertex

Course: Algebra II (Grades 9–12) **Subject:** Mathematics **Time:** ~50 min standard · ~90 min block **Work mode:** Independent · pencil + packet Math

Start Here

This is a **paper math packet** about **quadratic functions** — functions whose graphs are **parabolas** (U-shaped curves). You will read a quadratic written in **standard form** $y = ax^2 + bx + c$, find its **vertex** (the turning point) and its **axis of symmetry**, find its **zeros (roots)** by **factoring**, and **interpret the vertex** as a maximum or minimum in a real situation. Work by yourself with a pencil. **A calculator is not required, but a graphing or scientific calculator is allowed** if you have one. **Show your work** — every answer should show the numbers and steps you used, not just a final number. Everything you need (formulas, a worked example, a graph) is printed right here. If a question is hard, skip it, keep going, and come back. Circle or box your final answers.

Start Warm-Up: Retrieval 6 min

Bring back what you already know about evaluating and factoring.

1Evaluate a quadratic. A function is defined by $f(x) = x^2 - 4x + 3$. Find $f(0)$ and $f(2)$. Show the squaring, the multiplication, and the addition you used. (Remember: square the input first, then multiply, then combine.)

2Factor and use the Zero Product Property. Factor $x^2 + 5x + 6$ into two binomials (find two numbers that multiply to 6 and add to 5). Then find the two values of x that make the product equal 0.

Build Study the Math 8–12 min

What you need to know (with formulas)

A **quadratic function** has a graph that is a **parabola** — a symmetric U-shaped curve. Its **standard form** is:

$$y = ax^2 + bx + c \quad (a \neq 0)$$

The number **a** controls the opening. If $a > 0$ the parabola **opens up** and its vertex is the **lowest** point (a **minimum**). If $a < 0$ it **opens down** and its vertex is the **highest** point (a **maximum**). The number **c** is the **y-intercept** — the output when $x = 0$, the point $(0, c)$.

Vertex. The **vertex** is the turning point of the parabola. Its x -coordinate is found from a and b :

$$x_{\text{vertex}} = -b \div (2a)$$

Then substitute that x back into the function to get the y -coordinate. The vertex is the point $(x_{\text{vertex}}, y_{\text{vertex}})$.

Axis of symmetry. The parabola is a mirror image across the **vertical line** through the vertex:

$$x = -b \div (2a) \quad (\text{a vertical line})$$

Two points with the **same output** sit at the **same distance** on each side of this line. That is why the two zeros (below) are equally spaced around the axis.

Zeros (roots). The **zeros** (also called **roots** or **x-intercepts**) are the x-values where the output is **0** — where the parabola crosses the x-axis. To find them by **factoring**, set $y = 0$ and factor:

$$ax^2 + bx + c = 0 \rightarrow (x - r_1)(x - r_2) = 0 \rightarrow x = r_1 \text{ or } x = r_2$$

This uses the **Zero Product Property**: if two factors multiply to 0, then at least one factor must be 0. Set **each factor equal to 0** and solve.

Vertex as max or min in context. In a real story, the vertex often answers "how high" or "how much." For a thrown object, the vertex is the **greatest height** (a maximum): the x-coordinate tells **when** and the y-coordinate tells **how high**. For an area or cost problem the vertex can be a **maximum area** or a **minimum cost**.

Tip: the axis of symmetry $x = -b \div (2a)$ and the vertex share the **same** x-value. Find x first; the axis is that vertical line and the vertex is that point on the curve.

Worked Example — $y = x^2 - 4x + 3$

Here $a = 1$, $b = -4$, $c = 3$. Because $a = 1 > 0$, the parabola **opens up**, so the vertex is a **minimum**. The **y-intercept** is $c = 3$, the point $(0, 3)$.

Axis of symmetry / vertex x: $x = -b \div (2a) = -(-4) \div (2 \cdot 1) = 4 \div 2 = 2$. So the axis of symmetry is the vertical line $x = 2$.

Vertex y: substitute $x = 2$: $f(2) = (2)^2 - 4(2) + 3 = 4 - 8 + 3 = -1$. The **vertex is $(2, -1)$** — the lowest point.

Zeros by factoring: set $x^2 - 4x + 3 = 0$. Two numbers that multiply to +3 and add to -4 are -1 and -3, so it factors as $(x - 1)(x - 3) = 0$. By the Zero Product Property, $x - 1 = 0$ or $x - 3 = 0$, giving $x = 1$ or $x = 3$. The parabola crosses the x-axis at $(1, 0)$ and $(3, 0)$.

Symmetry check: the two zeros 1 and 3 are the same distance (1 unit) on each side of the axis $x = 2$. The midpoint of the zeros, $(1 + 3) \div 2 = 2$, is the axis. ✓

Outputs of $f(x) = x^2 - 4x + 3$ (notice the symmetry around $x = 2$).

x	f(x)	Note
0	3	y-intercept $(0, 3)$
1	0	zero / x-intercept
2	-1	vertex (minimum)
3	0	zero / x-intercept
4	3	mirror of $(0, 3)$

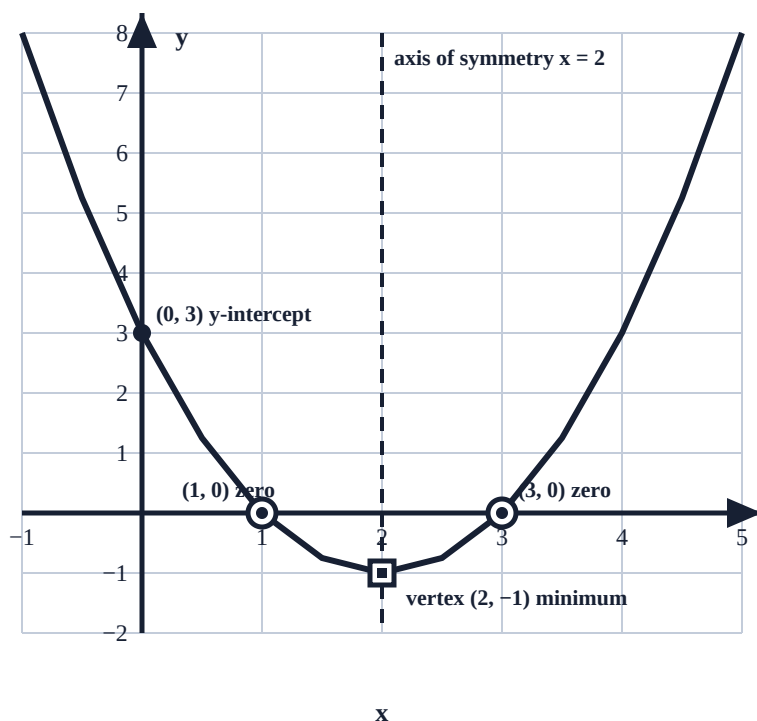


Figure 1. The graph of $y = x^2 - 4x + 3$. The parabola **opens up** ($a = 1 > 0$), so its vertex **(2, -1)** is a **minimum**. The dashed vertical line is the **axis of symmetry $x = 2$** . The curve crosses the x-axis at the two zeros **(1, 0)** and **(3, 0)** and crosses the y-axis at **(0, 3)**. Each plotted point satisfies the equation (for example $f(1) = 0$, $f(2) = -1$, $f(3) = 0$). The curve, points, and labels are drawn in black with text so the figure reads clearly in grayscale.

3Using the worked example graph, find **$f(4)$** for $f(x) = x^2 - 4x + 3$, and explain in one sentence why $f(4)$ has the **same** value as $f(0)$. (Hint: use the axis of symmetry $x = 2$.)

Apply Use What You Know 20–26 min

Show the numbers you use in every item. A calculator is allowed.

4**Read a graph.** A parabola is described below. It opens up and passes through the points $(-1, 0)$, $(0, -3)$, $(1, -4)$, $(2, -3)$, and $(3, 0)$. Use the description to answer 4a–4d.

Table
for Item
4.

Points
on the
parabola
(opens
up).

x	y
-1	0
0	-3
1	-4
2	-3
3	0

4a. Name the two **zeros (x-intercepts)** — the x-values where $y = 0$.

4b. Give the **vertex** (the lowest point in the table) as an ordered pair, and state whether it is a **maximum** or a **minimum**.

4c. Give the **axis of symmetry** as an equation $x = \underline{\hspace{2cm}}$. Explain in one sentence how the two zeros show you the axis.

4d. What is the **y-intercept** (the value of y when $x = 0$)?

5Factor to find the roots. Solve $x^2 - 7x + 10 = 0$ by factoring. Show the two numbers you found (they multiply to 10 and add to -7), the factored form, and each root from the Zero Product Property. Then state the **axis of symmetry** (halfway between the roots).

6Match a quadratic to a context. A soccer ball is kicked and its height in feet after t seconds is $h(t) = -16t^2 + 32t$ (here $a = -16$, $b = 32$, $c = 0$).

6a. Does this parabola open **up** or **down**? Is the vertex a **maximum** or a **minimum**? Answer in one sentence, using the sign of a .

6b. Find the **time** of the vertex using $t = -b \div (2a)$, then find the **greatest height** by evaluating h at that time. Show your steps and interpret the vertex in the story (when is the ball highest, and how high?).

7Error analysis (a sign error in factoring). A student solved $x^2 - x - 6 = 0$ and wrote the work shown. **Find the mistake, explain it, and give the correct roots.**

Student's work:

$$x^2 - x - 6 = (x - 2)(x - 3) = 0$$

So $x - 2 = 0$ or $x - 3 = 0$, giving $x = 2$ or $x = 3$.

What is wrong, and what are the correct roots? Fix it in the box.

Block Extra Applied Task block only • ~12–15 min

If your class is on a ~90-minute block schedule, complete Item 8. (On a standard ~50-minute period, this item is optional.)

8Vertex form and a second model. The **vertex form** of a quadratic is $y = a(x - h)^2 + k$, where the vertex is the point **(h, k)** and the axis of symmetry is $x = h$. This form shows the vertex directly, with no calculation needed.

8a. A parabola is given by $y = 2(x - 3)^2 - 5$. Write its **vertex** (h , k) and its **axis of symmetry**. Does it open up or down, and is the vertex a max or a min?

8b. Area modeling. A rancher has **40 feet** of fence to enclose a rectangular pen against a barn wall (the wall is one side, so the fence covers the other **three** sides). If the width is x , the two widths and one length use $2x + \text{length} = 40$, so $\text{length} = 40 - 2x$, and the area is $A(x) = x(40 - 2x) = -2x^2 + 40x$. Find the width x at the **vertex** (use $x = -b \div (2a)$), then the **maximum area**. Show your steps and interpret the vertex (what width gives the biggest pen, and how big is it?).

Explain Justify Your Strategy 6–10 min

Question 9. A classmate says, "For the quadratic $y = x^2 - 6x + 8$, the vertex is at $x = 4$ because the two roots are 2 and 4." **Is the classmate right? Find the true roots and the true vertex, and decide.** Write a **Claim** (your answer), **Evidence** (the roots you found by factoring and the vertex x from $-b \div (2a)$), and **Reasoning** (why the axis of symmetry sits **halfway between** the roots, not **at** a root).

Sentence stems you may use: "The classmate is ___ because..." · "Factoring $x^2 - 6x + 8$ gives... so the roots are..." · "The axis of symmetry is $x = -b \div (2a) = \dots$ " · "The vertex sits halfway between the roots because..." · "A root is where $y = 0$, but the vertex is where the curve turns, so..."

Claim (your answer / strategy)

Evidence (the numbers you used)

Reasoning (why the strategy works)

Close ACE Wrap-Up 5 min

Articulate

In your own words, explain the difference between a **zero (root)** of a quadratic and its **vertex**. Which one is where the curve crosses the x -axis, and which one is where the curve turns?

Connect

Point to one item in this packet where you found a **vertex** and used it as a **maximum or minimum** in a real context. Give its number.

Extend

Give a **new** real-life example where finding a **vertex** answers a useful question (a maximum or a minimum) that was *not* in this packet.

Continue Early Finisher optional

If you finish early (a challenge — no new materials needed):

Build your own quadratic. On the back of this page: (1) choose two whole-number roots you like (for example 2 and 5) and write a factored quadratic $(x - r_1)(x - r_2)$; (2) **multiply it out** into standard form $y = x^2 + bx + c$; (3) find the **axis of symmetry** and the **vertex**; and (4) make a small table of 5 outputs and sketch the parabola,

marking the vertex, the axis, and both zeros. Check that your two roots are the same distance on each side of your axis.

Turn in: Hand in this whole packet with your name, class period, and date filled in. Make sure items 1–7 and 9 and the ACE box are answered **with your work shown** (item 8 too if you are on a block schedule). The early-finisher challenge is optional but turn it in too if you did it.

HS_ALG2_Quadratics_01 — Quadratic Functions: Graphs, Zeros, and the Vertex Student Packet



TEACHER ANSWER KEY FOLLOWS

Do not copy the pages after this divider for students.

Answer Key: Quadratic Functions — Graphs, Zeros, and the Vertex

Course: Algebra II (Grades 9–12) **Subject:** Mathematics **For teacher use only** Math

How to use this key

Answers are grouped by section and question number, with full worked steps. Accept any correct method that reaches the right value — alternate strategies (factoring, completing a table, using $-b \div (2a)$, or reading symmetry from the graph) are noted where they apply. Watch-for notes flag common misconceptions. A graphing or scientific calculator is allowed, so grade the **reasoning and steps**, not just the arithmetic. Estimated grading time: ~6–8 min per packet.

Start Warm-Up: Retrieval ~1 min

1 Evaluate $f(x) = x^2 - 4x + 3$ at $x = 0$ and $x = 2$.

$f(0) = (0)^2 - 4(0) + 3 = 0 - 0 + 3 = 3$. $f(2) = (2)^2 - 4(2) + 3 = 4 - 8 + 3 = -1$. Full credit for squaring first, then multiplying, then combining. *Watch-for:* a student who computes $(-4 \cdot 2)$ before squaring correctly, or who writes $(2)^2 = 4$ but then does $4 - 4 = 0$ and forgets the -8 ; anchor the order: square, multiply, add/subtract left to right.

2 Factor $x^2 + 5x + 6$ and use the Zero Product Property.

Two numbers that multiply to 6 and add to 5 are 2 and 3, so $x^2 + 5x + 6 = (x + 2)(x + 3)$. Setting each factor to 0: $x + 2 = 0 \rightarrow x = -2$; $x + 3 = 0 \rightarrow x = -3$. *Watch-for:* sign confusion — because both middle and last terms are positive, both factors are $(x + \dots)$; the roots are negative.

Build Study the Math ~1 min

3 Find $f(4)$ for $f(x) = x^2 - 4x + 3$ and explain via symmetry.

$f(4) = (4)^2 - 4(4) + 3 = 16 - 16 + 3 = 3$. It equals $f(0) = 3$ because $x = 0$ and $x = 4$ are the **same distance (2 units)** on each side of the axis of symmetry $x = 2$; symmetric inputs have equal outputs. *Alternate:* read $(4, 3)$ as the mirror of $(0, 3)$ straight from the graph or table.

Apply Use What You Know ~3–4 min

4 Read a graph/table: points $(-1, 0)$, $(0, -3)$, $(1, -4)$, $(2, -3)$, $(3, 0)$.

4a. Zeros are where $y = 0$: $x = -1$ and $x = 3$ (the points $(-1, 0)$ and $(3, 0)$).

4b. Lowest point in the table is $(1, -4)$, so the **vertex is $(1, -4)$** . Because the parabola opens up, it is a **minimum**.

4c. Axis of symmetry: $x = 1$. The two zeros -1 and 3 are equally spaced around it — the midpoint $(-1 + 3) \div 2 = 1$ is the axis. (Also visible because $(0, -3)$ and $(2, -3)$ share a y -value and straddle $x = 1$.)

4d. y -intercept: $y = -3$ (the value at $x = 0$, the point $(0, -3)$).

Background (not required from students): this parabola is $y = x^2 - 2x - 3 = (x + 1)(x - 3)$; vertex $x = -(-2) \div 2 = 1$, $f(1) = 1 - 2 - 3 = -4$. ✓

5 Factor to solve $x^2 - 7x + 10 = 0$.

Two numbers that multiply to 10 and add to -7 are -2 and -5 : $x^2 - 7x + 10 = (x - 2)(x - 5) = 0$. Zero Product Property: $x - 2 = 0 \rightarrow x = 2$; $x - 5 = 0 \rightarrow x = 5$. Axis of symmetry is halfway between the roots: $x = (2 + 5) \div 2 = 3.5$. Check: $-b \div (2a) = 7 \div 2 = 3.5$. ✓

6 Match to context: $h(t) = -16t^2 + 32t$ (kicked ball).

6a. Since $a = -16 < 0$, the parabola **opens down**, so the vertex is a **maximum** (a highest point).

6b. Time of vertex: $t = -b \div (2a) = -32 \div (2 \cdot -16) = -32 \div -32 = 1$ **second**. Greatest height: $h(1) = -16(1)^2 + 32(1) = -16 + 32 = 16$ **feet**. **Interpretation:** the ball reaches its **highest point of 16 feet at 1 second** after the kick. *Alternate:* the zeros are $t = 0$ and $t = 2$ (factor $-16t(t - 2)$); their midpoint $t = 1$ is the vertex time. *Watch-for:* dropping the negative on a and getting the wrong sign, or stopping after finding $t = 1$ without evaluating the height.

7 Error analysis (sign error in factoring): $x^2 - x - 6 = 0$.

The mistake: The student factored as $(x - 2)(x - 3)$, but that **multiplies out to $x^2 - 5x + 6$** , not $x^2 - x - 6$. They ignored the **negative constant -6** : the two numbers must multiply to -6 (so they have **opposite signs**) and add to -1 .

Correct work: $-6 = (-3)(+2)$ and $-3 + 2 = -1$, so $x^2 - x - 6 = (x - 3)(x + 2) = 0$. Then $x - 3 = 0 \rightarrow x = 3$; $x + 2 = 0 \rightarrow x = -2$. The correct roots are **$x = 3$ and $x = -2$** .

Full credit requires (1) naming that the factors must multiply to -6 (opposite signs), not $+6$, and (2) the correct roots 3 and -2 . *Sense-check to offer:* substitute $x = 2$ into the original: $4 - 2 - 6 = -4 \neq 0$, so 2 is not a root; $x = 3$: $9 - 3 - 6 = 0$ ✓.

Block Extra Applied Task block only • ~1–2 min

8 Vertex form and a second model (block schedule).

8a. For $y = 2(x - 3)^2 - 5$, the form $y = a(x - h)^2 + k$ gives $h = 3$ and $k = -5$, so the **vertex is $(3, -5)$** and the **axis of symmetry is $x = 3$** . Since $a = 2 > 0$, it **opens up** and the vertex is a **minimum**. *Watch-for:* the sign of h — the form uses $(x - h)$, so $(x - 3)$ means $h = +3$, not -3 .

8b. $A(x) = -2x^2 + 40x$, so $a = -2$, $b = 40$. Vertex width: $x = -b \div (2a) = -40 \div (2 \cdot -2) = -40 \div -4 = 10$ **feet**. Maximum area: $A(10) = -2(10)^2 + 40(10) = -200 + 400 = 200$ **square feet**. **Interpretation:** a width of **10 ft** (with length $40 - 2(10) = 20$ ft) gives the **largest pen, 200 sq ft**. Because $a < 0$, the vertex is a **maximum**. *Alternate:* zeros of A are $x = 0$ and $x = 20$ (factor $-2x(x - 20)$); midpoint $x = 10$ is the vertex.

Explain Justify Your Strategy (Q9) ~1–2 min

9 "For $y = x^2 - 6x + 8$, the vertex is at $x = 4$ because the roots are 2 and 4." Right?

Model answer. Claim: The classmate is **wrong**. The roots are 2 and 4, but the vertex is at **$x = 3$** , not $x = 4$.

Evidence: Factor: $x^2 - 6x + 8 = (x - 2)(x - 4) = 0$, so the roots are **$x = 2$ and $x = 4$** . Axis/vertex $x = -b \div (2a) = -(-6) \div (2 \cdot 1) = 6 \div 2 = 3$; the vertex is $(3, f(3)) = (3, 9 - 18 + 8) = (3, -1)$. **Reasoning:** The axis of symmetry sits **halfway between the two roots**: $(2 + 4) \div 2 = 3$. A root is a point where $y = 0$ (the curve crosses the x -axis), but the vertex is where the curve **turns** — it is the midpoint of the two roots, not one of the roots themselves. The classmate mixed up a root ($x = 4$) with the axis ($x = 3$).

Note: Full credit requires the correct roots (2 and 4), the correct vertex x (3), and a clear statement that the axis is the **midpoint** of the roots — so the classmate is wrong.

Justification scoring rubric (3 points)

Score	Claim	Evidence	Reasoning
3	States the classmate is wrong AND that the vertex is at $x = 3$.	Roots 2 and 4 found by factoring AND vertex $x = 3$ from $-b \div (2a)$ (or midpoint), shown.	Explains the axis is the midpoint of the roots and distinguishes a root ($y = 0$) from the vertex (turning point).
2	Says the classmate is wrong but vertex value unclear or partly right.	Roots found, or vertex found, but not both clearly shown.	Reasoning present but incomplete (e.g., states $x = 3$ without the midpoint idea).
1	A judgment made with weak or no support.	Evidence largely missing or incorrect.	Little or flawed reasoning.
0	No/incorrect claim.	No evidence.	No reasoning.

Close ACE ~1 min

ACE Articulate / Connect / Extend.

Articulate: Accept any correct explanation: "A **zero (root)** is an x -value where $y = 0$ — where the parabola **crosses the x -axis**. The **vertex** is the single point where the curve **turns** (its lowest or highest point). A parabola can have two zeros but only one vertex; the zeros cross the axis, the vertex sits halfway between them."

Connect: Valid items where a vertex was used as a max/min in context include **Q6** (greatest height, a maximum) and, on block, **Q8b** (maximum area). Either earns credit if correctly named.

Extend: Any genuine max/min situation — e.g., the maximum revenue as ticket price changes, the minimum cost of production, the peak height of a fireworks shell, or the largest rectangular area for a fixed perimeter. Must clearly identify the quantity being maximized or minimized at the vertex.

Challenge Early Finisher (optional)

EF Build your own quadratic from two roots.

Sample (roots 2 and 5): factored form $(x - 2)(x - 5)$. **(2) Multiply out:** $x^2 - 7x + 10$, so $y = x^2 - 7x + 10$. **(3)**

Axis: $x = (2 + 5) \div 2 = 3.5$ (or $-b \div (2a) = 7 \div 2 = 3.5$); **vertex:** $f(3.5) = 12.25 - 24.5 + 10 = -2.25$, so $(3.5, -2.25)$. **(4)** A reasonable table: $(2, 0)$, $(3, -2)$, $(3.5, -2.25)$, $(4, -2)$, $(5, 0)$, symmetric around $x = 3.5$. *Full credit* requires a correct factored form, a correct standard form, an axis halfway between the two chosen roots, a matching vertex, and a symmetric table/sketch. Many correct answers exist depending on the roots chosen.

Watch Common misconceptions

- **Roots vs. vertex.** Students confuse a **zero** (where $y = 0$, on the x -axis) with the **vertex** (the turning point). A common error (Q9) is naming a root as the axis. Anchor: the axis of symmetry is the **midpoint of the two roots**, $x = (r_1 + r_2) \div 2$, and the vertex sits on that line — usually **not** at a root.
- **Sign errors in factoring.** The Q7 error: for $x^2 - x - 6$ the constant is **negative**, so the two numbers must have **opposite signs** and multiply to -6 . Students who force both factors the same sign (e.g., $(x - 2)(x - 3)$) get the wrong middle term. Anchor: check the sign of c — negative c means opposite-sign factors.
- **Axis of symmetry sign slips.** In $x = -b \div (2a)$, students drop the leading negative or mishandle a negative b (as in $b = -4$, where $-b = +4$). Encourage writing $-(-4)$ explicitly. In vertex form $y = a(x - h)^2 + k$, $(x - 3)$ means $h = +3$, not -3 (Q8a).
- **Ignoring the sign of a for max vs. min.** $a > 0$ opens up $\rightarrow$ minimum; $a < 0$ opens down $\rightarrow$ maximum (Q6 has $a = -16$, a maximum). Students who assume "vertex = minimum" always miss projectile-height maxima.
- **Stopping after finding the vertex x .** In Q6 and Q8b, after t or x is found the student must **substitute back** to get the height/area. Encourage a check by evaluating the function at the vertex x .

- **Order of operations in a quadratic.** In $f(x) = x^2 - 4x + 3$, students must **square first**, then multiply the $-4x$ term, then combine (Q1). A student who does $x - 4 = \dots$ before squaring has misread the expression.
- **Reading the y-intercept as a zero.** The y-intercept $(0, c)$ is where the curve crosses the **y-axis**, not a zero. In the worked example $c = 3$ gives $(0, 3)$, which is not a root; the roots are $(1, 0)$ and $(3, 0)$.

Teacher follow-up based on likely errors

If many students miss Q7, do a quick 5-minute practice factoring trinomials with a **negative constant**, stressing that the two numbers have opposite signs and multiply to c . If Q9 shows the "root equals axis" idea, re-anchor that the axis of symmetry is the **midpoint** of the roots and the vertex sits there (not at a root). The Q6 and Q8b items reveal who can interpret a vertex as a real-world **maximum** (height, area). The Q4 item shows who can read zeros, vertex, axis, and y-intercept from a graph/table — all good warm-up discussions next class. The separate answer key's rubric scores the Q9 justification.

HS_ALG2_Quadratics_01 — Quadratic Functions: Graphs, Zeros, and the Vertex Teacher Answer Key