

Answer Key: Quadratic Functions — Graphs, Zeros, and the Vertex

Course: Algebra II (Grades 9–12) **Subject:** Mathematics **For teacher use only** Math

How to use this key

Answers are grouped by section and question number, with full worked steps. Accept any correct method that reaches the right value — alternate strategies (factoring, completing a table, using $-b \div (2a)$, or reading symmetry from the graph) are noted where they apply. Watch-for notes flag common misconceptions. A graphing or scientific calculator is allowed, so grade the **reasoning and steps**, not just the arithmetic. Estimated grading time: ~6–8 min per packet.

Start Warm-Up: Retrieval ~1 min

1 Evaluate $f(x) = x^2 - 4x + 3$ at $x = 0$ and $x = 2$.

$f(0) = (0)^2 - 4(0) + 3 = 0 - 0 + 3 = \mathbf{3}$. $f(2) = (2)^2 - 4(2) + 3 = 4 - 8 + 3 = \mathbf{-1}$. Full credit for squaring first, then multiplying, then combining. *Watch-for:* a student who computes $(-4 \cdot 2)$ before squaring correctly, or who writes $(2)^2 = 4$ but then does $4 - 4 = 0$ and forgets the -8 ; anchor the order: square, multiply, add/subtract left to right.

2 Factor $x^2 + 5x + 6$ and use the Zero Product Property.

Two numbers that multiply to 6 and add to 5 are 2 and 3, so $x^2 + 5x + 6 = \mathbf{(x + 2)(x + 3)}$. Setting each factor to 0: $x + 2 = 0 \rightarrow x = \mathbf{-2}$; $x + 3 = 0 \rightarrow x = \mathbf{-3}$. *Watch-for:* sign confusion — because both middle and last terms are positive, both factors are $(x + \dots)$; the roots are negative.

Build Study the Math ~1 min

3 Find $f(4)$ for $f(x) = x^2 - 4x + 3$ and explain via symmetry.

$f(4) = (4)^2 - 4(4) + 3 = 16 - 16 + 3 = \mathbf{3}$. It equals $f(0) = 3$ because $x = 0$ and $x = 4$ are the **same distance (2 units)** on each side of the axis of symmetry $x = 2$; symmetric inputs have equal outputs. *Alternate:* read $(4, 3)$ as the mirror of $(0, 3)$ straight from the graph or table.

Apply Use What You Know ~3–4 min

4 Read a graph/table: points $(-1, 0)$, $(0, -3)$, $(1, -4)$, $(2, -3)$, $(3, 0)$.

4a. Zeros are where $y = 0$: $x = \mathbf{-1}$ and $x = \mathbf{3}$ (the points $(-1, 0)$ and $(3, 0)$).

4b. Lowest point in the table is $(1, -4)$, so the **vertex is $(1, -4)$** . Because the parabola opens up, it is a **minimum**.

4c. Axis of symmetry: $x = \mathbf{1}$. The two zeros -1 and 3 are equally spaced around it — the midpoint $(-1 + 3) \div 2 = 1$ is the axis. (Also visible because $(0, -3)$ and $(2, -3)$ share a y -value and straddle $x = 1$.)

4d. y -intercept: $y = \mathbf{-3}$ (the value at $x = 0$, the point $(0, -3)$).

Background (not required from students): this parabola is $y = x^2 - 2x - 3 = (x + 1)(x - 3)$; vertex $x = -(-2) \div 2 = 1$, $f(1) = 1 - 2 - 3 = -4$. ✓

5 Factor to solve $x^2 - 7x + 10 = 0$.

Two numbers that multiply to 10 and add to -7 are -2 and -5 : $x^2 - 7x + 10 = (x - 2)(x - 5) = 0$. Zero Product Property: $x - 2 = 0 \rightarrow x = 2$; $x - 5 = 0 \rightarrow x = 5$. Axis of symmetry is halfway between the roots: $x = (2 + 5) \div 2 = 3.5$. Check: $-b \div (2a) = 7 \div 2 = 3.5$. ✓

6 Match to context: $h(t) = -16t^2 + 32t$ (kicked ball).

6a. Since $a = -16 < 0$, the parabola **opens down**, so the vertex is a **maximum** (a highest point).

6b. Time of vertex: $t = -b \div (2a) = -32 \div (2 \cdot -16) = -32 \div -32 = 1$ second. Greatest height: $h(1) = -16(1)^2 + 32(1) = -16 + 32 = 16$ feet. Interpretation: the ball reaches its **highest point of 16 feet at 1 second** after the kick. Alternate: the zeros are $t = 0$ and $t = 2$ (factor $-16t(t - 2)$); their midpoint $t = 1$ is the vertex time. Watch-for: dropping the negative on a and getting the wrong sign, or stopping after finding $t = 1$ without evaluating the height.

7 Error analysis (sign error in factoring): $x^2 - x - 6 = 0$.

The mistake: The student factored as $(x - 2)(x - 3)$, but that **multiplies out to $x^2 - 5x + 6$** , not $x^2 - x - 6$. They ignored the **negative constant -6** : the two numbers must multiply to -6 (so they have **opposite signs**) and add to -1 .

Correct work: $-6 = (-3)(+2)$ and $-3 + 2 = -1$, so $x^2 - x - 6 = (x - 3)(x + 2) = 0$. Then $x - 3 = 0 \rightarrow x = 3$; $x + 2 = 0 \rightarrow x = -2$. The correct roots are **$x = 3$ and $x = -2$** .

Full credit requires (1) naming that the factors must multiply to -6 (opposite signs), not $+6$, and (2) the correct roots 3 and -2 . Sense-check to offer: substitute $x = 2$ into the original: $4 - 2 - 6 = -4 \neq 0$, so 2 is not a root; $x = 3$: $9 - 3 - 6 = 0$ ✓.

Block Extra Applied Task block only • ~1–2 min

8 Vertex form and a second model (block schedule).

8a. For $y = 2(x - 3)^2 - 5$, the form $y = a(x - h)^2 + k$ gives $h = 3$ and $k = -5$, so the **vertex is $(3, -5)$** and the **axis of symmetry is $x = 3$** . Since $a = 2 > 0$, it **opens up** and the vertex is a **minimum**. Watch-for: the sign of h — the form uses $(x - h)$, so $(x - 3)$ means $h = +3$, not -3 .

8b. $A(x) = -2x^2 + 40x$, so $a = -2$, $b = 40$. Vertex width: $x = -b \div (2a) = -40 \div (2 \cdot -2) = -40 \div -4 = 10$ feet. Maximum area: $A(10) = -2(10)^2 + 40(10) = -200 + 400 = 200$ square feet. Interpretation: a width of **10 ft** (with length $40 - 2(10) = 20$ ft) gives the **largest pen, 200 sq ft**. Because $a < 0$, the vertex is a **maximum**. Alternate: zeros of A are $x = 0$ and $x = 20$ (factor $-2x(x - 20)$); midpoint $x = 10$ is the vertex.

Explain Justify Your Strategy (Q9) ~1–2 min

9 "For $y = x^2 - 6x + 8$, the vertex is at $x = 4$ because the roots are 2 and 4." Right?

Model answer. Claim: The classmate is **wrong**. The roots are 2 and 4, but the vertex is at **$x = 3$** , not $x = 4$.

Evidence: Factor: $x^2 - 6x + 8 = (x - 2)(x - 4) = 0$, so the roots are **$x = 2$ and $x = 4$** . Axis/vertex $x = -b \div (2a) = -(-6) \div (2 \cdot 1) = 6 \div 2 = 3$; the vertex is $(3, f(3)) = (3, 9 - 18 + 8) = (3, -1)$. **Reasoning:** The axis of symmetry sits **halfway between the two roots**: $(2 + 4) \div 2 = 3$. A root is a point where $y = 0$ (the curve crosses the x -axis), but the vertex is where the curve **turns** — it is the midpoint of the two roots, not one of the roots themselves. The classmate mixed up a root ($x = 4$) with the axis ($x = 3$).

Note: Full credit requires the correct roots (2 and 4), the correct vertex x (3), and a clear statement that the axis is the **midpoint** of the roots — so the classmate is wrong.

Justification scoring rubric (3 points)

Score	Claim	Evidence	Reasoning
3	States the classmate is wrong AND that the vertex is at $x = 3$.	Roots 2 and 4 found by factoring AND vertex $x = 3$ from $-b \div (2a)$ (or midpoint), shown.	Explains the axis is the midpoint of the roots and distinguishes a root ($y = 0$) from the vertex (turning point).
2	Says the classmate is wrong but vertex value unclear or partly right.	Roots found, or vertex found, but not both clearly shown.	Reasoning present but incomplete (e.g., states $x = 3$ without the midpoint idea).
1	A judgment made with weak or no support.	Evidence largely missing or incorrect.	Little or flawed reasoning.
0	No/incorrect claim.	No evidence.	No reasoning.

Close ACE ~1 min

ACE Articulate / Connect / Extend.

Articulate: Accept any correct explanation: "A **zero (root)** is an x -value where $y = 0$ — where the parabola **crosses the x -axis**. The **vertex** is the single point where the curve **turns** (its lowest or highest point). A parabola can have two zeros but only one vertex; the zeros cross the axis, the vertex sits halfway between them."

Connect: Valid items where a vertex was used as a max/min in context include **Q6** (greatest height, a maximum) and, on block, **Q8b** (maximum area). Either earns credit if correctly named.

Extend: Any genuine max/min situation — e.g., the maximum revenue as ticket price changes, the minimum cost of production, the peak height of a fireworks shell, or the largest rectangular area for a fixed perimeter. Must clearly identify the quantity being maximized or minimized at the vertex.

Challenge Early Finisher (optional)

EF Build your own quadratic from two roots.

Sample (roots 2 and 5): factored form $(x - 2)(x - 5)$. **(2) Multiply out:** $x^2 - 7x + 10$, so $y = x^2 - 7x + 10$. **(3)**

Axis: $x = (2 + 5) \div 2 = 3.5$ (or $-b \div (2a) = 7 \div 2 = 3.5$); **vertex:** $f(3.5) = 12.25 - 24.5 + 10 = -2.25$, so $(3.5, -2.25)$. **(4)** A reasonable table: $(2, 0)$, $(3, -2)$, $(3.5, -2.25)$, $(4, -2)$, $(5, 0)$, symmetric around $x = 3.5$. *Full credit* requires a correct factored form, a correct standard form, an axis halfway between the two chosen roots, a matching vertex, and a symmetric table/sketch. Many correct answers exist depending on the roots chosen.

Watch Common misconceptions

- **Roots vs. vertex.** Students confuse a **zero** (where $y = 0$, on the x -axis) with the **vertex** (the turning point). A common error (Q9) is naming a root as the axis. Anchor: the axis of symmetry is the **midpoint of the two roots**, $x = (r_1 + r_2) \div 2$, and the vertex sits on that line — usually **not** at a root.
- **Sign errors in factoring.** The Q7 error: for $x^2 - x - 6$ the constant is **negative**, so the two numbers must have **opposite signs** and multiply to -6 . Students who force both factors the same sign (e.g., $(x - 2)(x - 3)$) get the wrong middle term. Anchor: check the sign of c — negative c means opposite-sign factors.
- **Axis of symmetry sign slips.** In $x = -b \div (2a)$, students drop the leading negative or mishandle a negative b (as in $b = -4$, where $-b = +4$). Encourage writing $-(-4)$ explicitly. In vertex form $y = a(x - h)^2 + k$, $(x - 3)$ means $h = +3$, not -3 (Q8a).
- **Ignoring the sign of a for max vs. min.** $a > 0$ opens up $\rightarrow$ minimum; $a < 0$ opens down $\rightarrow$ maximum (Q6 has $a = -16$, a maximum). Students who assume "vertex = minimum" always miss projectile-height maxima.
- **Stopping after finding the vertex x .** In Q6 and Q8b, after t or x is found the student must **substitute back** to get the height/area. Encourage a check by evaluating the function at the vertex x .

- **Order of operations in a quadratic.** In $f(x) = x^2 - 4x + 3$, students must **square first**, then multiply the $-4x$ term, then combine (Q1). A student who does $x - 4 = \dots$ before squaring has misread the expression.
- **Reading the y-intercept as a zero.** The y-intercept $(0, c)$ is where the curve crosses the **y-axis**, not a zero. In the worked example $c = 3$ gives $(0, 3)$, which is not a root; the roots are $(1, 0)$ and $(3, 0)$.

Teacher follow-up based on likely errors

If many students miss Q7, do a quick 5-minute practice factoring trinomials with a **negative constant**, stressing that the two numbers have opposite signs and multiply to c . If Q9 shows the "root equals axis" idea, re-anchor that the axis of symmetry is the **midpoint** of the roots and the vertex sits there (not at a root). The Q6 and Q8b items reveal who can interpret a vertex as a real-world **maximum** (height, area). The Q4 item shows who can read zeros, vertex, axis, and y-intercept from a graph/table — all good warm-up discussions next class. The separate answer key's rubric scores the Q9 justification.

HS_ALG2_Quadratics_01 — Quadratic Functions: Graphs, Zeros, and the Vertex Teacher Answer Key