

Substitute Guide: Linear Relationships in Four Forms

Grade: 8 **Subject:** Mathematics **Time:** ~45 min core + ~15 min extension Math

Learning goal (plain language)

Students practice **linear relationships** — connecting the same "straight-line" relationship across a **table**, a **graph**, an **equation** ($y = mx + b$), and a **real-world story**. They find the **slope** (the rate, m), the **y-intercept** (the starting amount, b), and tell whether a relationship is **proportional** (through the origin) or **not**. **You do not need a math background to run this.** Every formula and a worked example are printed in the student packet. Students **show their work**; you do not need to teach the method — the packet does the teaching. Your job is to hand it out, read the start script, keep the room on task, and collect the packets.

Standards alignment

Framework: 19 TAC §111.28 (Grade 8 Mathematics). Knowledge & skills **8.4** and **8.5** — proportional and non-proportional linear relationships, slope, and y-intercept.

- **Primary — 8.4A:** use similar right triangles / rates to develop the idea of slope as a constant rate of change (rise over run).
- **Primary — 8.4C:** use data to write an equation in the form $y = mx + b$ and interpret slope (m) and y-intercept (b) in context.
- **Primary — 8.5I:** distinguish proportional ($y = kx$, through the origin) from non-proportional ($y = mx + b$, $b \neq 0$) linear relationships across tables, graphs, and equations.

Provisional — pending educator verification against the current official TAC source.

(Standards are paraphrased here, not quoted. Please confirm codes and wording against your official source.)

Setup Before class (5 min)

- Count and hand out one **student packet** per student.
- Confirm each student has a **pencil**. A **basic calculator is allowed but not required**; students who have one may use it.
- Write the date on the board and remind students to fill in the name line.
- Keep the **answer key** with you — it is a separate file and is **not** in the student packet.

Materials What's needed

- Printed student packet (4–6 pages) per student.
- Pencil per student; a basic four-function calculator is optional.
- This guide and the separate answer key (teacher only).
- No technology, no internet, no special supplies.

Timing Suggested pacing (~45–60 min)

Segment	Time	What students do
Start — Retrieval warm-up	5 min	Q1–Q2 (evaluate a rule; read a point off the line)
Build — Study the math	5–10 min	Read reference box, worked example, graph; Q3
Apply — Use what you know	20–25 min	Q4–Q6 (four forms, error analysis, predict from model)
Explain — Justify (CER)	5–10 min	Q7 claim/evidence/reasoning
Close — ACE	5 min	Articulate / Connect / Extend
Continue — early finishers	optional ~15 min	Write a context for $y = 8x + 15$

Script Read aloud to start

"Today's assignment is a paper math packet called *Linear Relationships in Four Forms*. You will work by yourself with a pencil. A basic calculator is allowed if you have one, but you do not need one. The most important rule: **show your work** — write down the numbers you use, not just a final answer. Everything you need is printed in the packet, including a worked example, a graph, and all the formulas. Start with the warm-up, then read the Build section before the Apply questions. If a question is tricky, skip it and come back. Put your name, class period, and date at the top now. You have about 45 minutes; raise your hand if you need help reading a question."

Support When students ask for help

- **Students show their work; you do not need to teach the method.** Point them back to the printed **reference box** and the **worked example** — the method is right there.
- **You may read any part aloud** to a student or the class.
- If a student is stuck, prompt with "What does the worked example do?" or "In $y = mx + b$, which number is the rate and which is the starting amount?" rather than giving the answer.
- For slope, remind them the packet shows: $\text{slope} = \text{rise} \div \text{run} = (\text{change in } y) \div (\text{change in } x)$, with the y -difference on top.
- It is fine if a student cannot finish the optional early-finisher challenge.

Access Accommodations & language support

- **Read-aloud** of any question is allowed for all students.
- Point students to the **worked example**, the **formulas** in the reference box, and the **sentence stems** for Q7 and ACE.
- A **calculator is allowed** for any student who wants one — this removes the arithmetic barrier so students can focus on the reasoning.
- Extended time is fine; the core can stop after Q7 if time runs short.
- Students may show work with numbers and short phrases if writing full sentences is a barrier, as long as the reasoning is clear.
- The packet is grayscale-safe — the graph uses black lines, labeled points, and text, so it prints and reads clearly in black and white.

Tools Allowed & not allowed

Allowed: pencil, the printed packet, a basic (four-function) calculator, quiet self-read-aloud. **Not needed / not allowed:** phones, internet, graphing/AI calculators, or getting answers from another student.

Collect at the end: Collect every packet, whether finished or not, with names on them. Stack them for the teacher. Note on a sticky how far most students got and any questions that caused confusion.

Answer key: The **answer key is a separate file** (answerkey.html) and is for the teacher only. Please do not hand it to students.

Teacher follow-up (for the returning teacher)

The highest-value items to review next class are the **error analysis (Q5)** and the **justification (Q7)**, which reveal the reversed rise/run slope error and whether students can compare two linear models using both slope and y-intercept. Watch for **slope vs. y-intercept swaps** on the four-forms item (Q4) and the belief that **every line is proportional** (Q4d, Q7 check this — only lines through the origin are). The separate answer key lists worked steps, alternate strategies, and a 3-point rubric for Q7.

G08_MATH_Linear_01 — Linear Relationships in Four Forms Substitute Guide

Linear Relationships in Four Forms

Grade: 8 **Subject:** Mathematics **Time:** ~45 min core + ~15 min extension **Work mode:** Independent · pencil + packet Math

Start Here

This is a **paper math packet** about **linear relationships** — the kind that make a straight line. You will connect the same relationship shown four ways: a **table**, a **graph**, an **equation**, and a **real-world story**. Work by yourself with a pencil. **A calculator is not required, but a basic calculator is allowed** if you have one. **Show your work** — every answer should show the numbers you used, not just a final number. Everything you need (formulas, a worked example, a graph) is printed right here. If a question is hard, skip it, keep going, and come back. Circle or box your final answers.

Start Warm-Up: Retrieval 5 min

Bring back what you already know about rules and reading a graph.

1Evaluate a rule. A rule says $y = 2x + 5$. Find the value of y when $x = 4$. Show the multiplication and the addition you used.

2Read a point off a line. On the graph in the Build section (Figure 1), the line passes through several marked points. Find the point where $x = 2$ and write it as an ordered pair (x, y) . (You may look ahead to Figure 1 to answer this.)

Build Study the Math 5–10 min

What you need to know (with formulas)

A **linear relationship** between two quantities x and y makes a **straight line** when graphed. It changes by the **same amount** each step.

The **rate of change** (also called the **slope**, written **m**) tells how much y changes for each 1 that x increases. From two points (x_1, y_1) and (x_2, y_2) :

$$m = \text{rise} \div \text{run} = (y_2 - y_1) \div (x_2 - x_1)$$

The **y-intercept** (written **b**) is the value of y when $x = 0$ — the point **$(0, b)$** where the line crosses the vertical axis. Together they give the **slope-intercept form** of a line:

$$y = mx + b \quad (m = \text{slope}, b = \text{y-intercept})$$

A linear relationship can be shown **four ways**, and all four match the same m and b :

- **Table:** as x goes up by a fixed step, y goes up (or down) by a fixed step. $m = (\text{change in } y) \div (\text{change in } x)$. The row where $x = 0$ gives b .
- **Graph:** a straight line. It crosses the y -axis at $(0, b)$ and rises m units for each 1 unit right.
- **Equation:** $y = mx + b$.

- **Situation:** a real story with a **starting amount** (b) and a steady **rate** (m) per one.

Proportional vs. non-proportional. A linear relationship is **proportional** only when $b = 0$ — the line passes through the origin $(0, 0)$ and $y = mx$. If b is **not 0** (there is a starting amount), the line is **linear but NOT proportional**; it crosses the y -axis above or below the origin.

Tip: not every straight line is proportional. Check the y -intercept — only a line through $(0, 0)$ is a proportional relationship.

Worked Example — one relationship, four forms

A pool already holds some water, then a hose adds water at a steady rate. The depth starts at **4 inches** and rises **3 inches every minute**. Because there is a **starting amount of 4**, this is **linear but NOT proportional**.

Slope (rate of change): $m = 3$ inches per minute. **y-intercept:** $b = 4$ inches (the depth at 0 minutes).

Equation: $y = 3x + 4$, where x = minutes and y = depth in inches.

Check the slope from two table rows: using $(1, 7)$ and $(2, 10)$, $m = (10 - 7) \div (2 - 1) = 3 \div 1 = 3$. ✓

Table (each step of 1 in x adds 3 to y) and **graph** below. The line crosses the y -axis at $(0, 4)$ — **not** the origin — so it is non-proportional.

Water depth over time ($y = 3x + 4$).

Time x (minutes) Depth y (inches) Change in y

0	4	— (start)
1	7	+3
2	10	+3
3	13	+3
4	16	+3

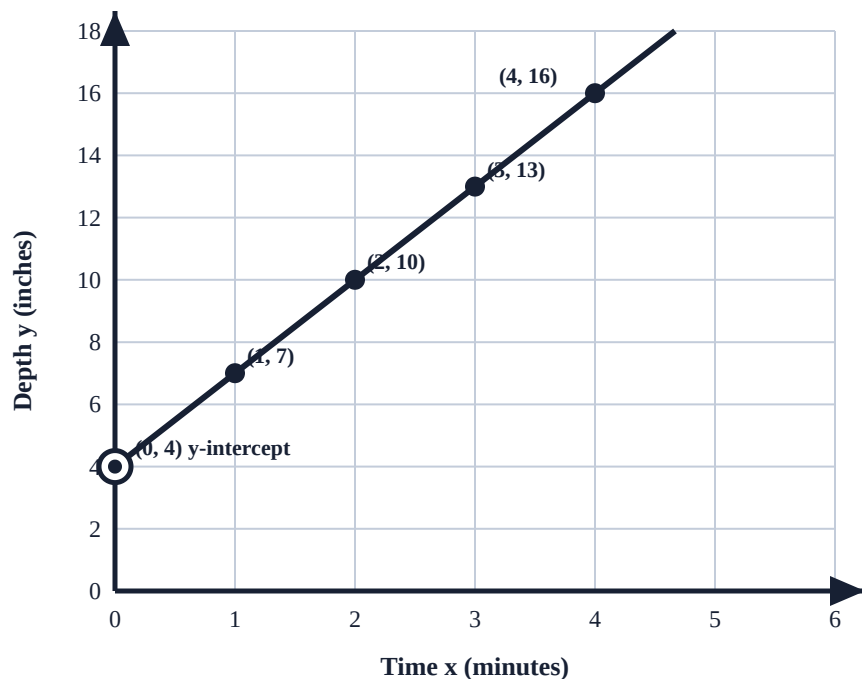


Figure 1. The graph of $y = 3x + 4$. It is a straight line that crosses the vertical axis at **(0, 4)** — not at the origin — so it is **linear but not proportional**. Each printed point (1, 7), (2, 10), (3, 13), (4, 16) matches a table row and rises 3 inches for every 1 minute. The line and points are drawn in black with text labels so the figure reads clearly in grayscale.

3Using the worked example, how deep would the water be at **6 minutes**? Use $y = 3x + 4$ and show your work.

Apply Use What You Know 20–25 min

Show the numbers you use in every item. A basic calculator is allowed.

4**Match the four forms.** A gym charges a one-time sign-up fee plus a fixed price for each visit. The **same** linear relationship is shown below as a table, a graph description, an equation, and a story. Study Table B, then answer 4a–4d.

Table B. Total cost of a gym membership (a linear relationship).

Visits x Total cost y (dollars)

0	20
2	30
4	40
6	50

Story: "A gym charges a one-time **\$20 sign-up fee**, then **\$5 for each visit**. Total cost = sign-up fee + \$5 times the number of visits."

4a. Find the **slope** m (dollars per visit). Use two rows of the table and show $m = (\text{change in } y) \div (\text{change in } x)$.

4b. Find the **y-intercept** b (the cost at 0 visits), then write the equation $y = mx + b$ for this relationship.

4c. **Interpret in context.** In this story, what does the **slope** mean, and what does the **y-intercept** mean? Use the words "per visit" and "sign-up fee."

4d. Is this relationship **proportional** or **not proportional**? Explain using the y-intercept in one sentence.

5**Error analysis (a mis-found slope).** A student found the slope for Table B and wrote the work shown. **Find the mistake, explain it, and give the correct slope.**

Student's work (using the rows (2, 30) and (4, 40)):

$$m = \text{run} \div \text{rise} = (4 - 2) \div (40 - 30)$$

$$m = 2 \div 10 = 0.2. \text{ So the slope is 0.2 dollars per visit.}$$

What is wrong, and what is the correct slope? Fix it in the box.

6**Multi-step application (predict a value from the model).** Use the gym relationship $y = 5x + 20$ from item 4.

6a. Predict the **total cost of 12 visits**. Show $y = 5(12) + 20$.

6b. A member paid a total of \$75. How many **visits** did they make? Set up $5x + 20 = 75$ and solve for x . Show your steps.

Explain Justify Your Strategy 5–10 min

Question 7. Two plans charge for renting a bike. **Plan A:** $y = 4x + 10$ (a \$10 sign-up plus \$4 per hour). **Plan B:** $y = 6x$ (just \$6 per hour, no sign-up). **For a 3-hour rental, which plan costs less, and why?** Also state which plan is **proportional**. Write a **Claim** (your answer), **Evidence** (the numbers you calculated), and **Reasoning** (why your strategy works, using slope and y-intercept).

Sentence stems you may use: "For 3 hours, Plan ____ costs less because..." · "I found Plan A's cost by... and Plan B's cost by..." · "Plan B is proportional because its y-intercept is..." · "The slope tells me... and the y-intercept tells me..."

Claim (your answer / strategy)

Evidence (the numbers you used)

Reasoning (why the strategy works)

Close ACE Wrap-Up 5 min

Articulate

In your own words, explain how to find the **slope** from two rows of a table using rise over run.

Connect

Point to one item in this packet where you used $y = mx + b$. Give its number.

Extend

Give a **new** real-life example of a **non-proportional** linear relationship (a starting amount plus a steady rate) that was *not* in this packet.

Continue Early Finisher optional · ~15 min

If you finish early (a challenge — no new materials needed):

Write a context for an equation. Here is an equation with no story: $y = 8x + 15$. On the back of this page: (1) invent a **real-world situation** that this equation could describe, clearly telling what x and y stand for; (2) explain what the **slope 8** and the **y-intercept 15** mean in your story; and (3) use your model to **predict y when $x = 5$** , showing your work. Make sure your story is **non-proportional** (it must have a starting amount).

Turn in: Hand in this whole packet with your name, class period, and date filled in. Make sure questions 1–7 and the ACE box are answered **with your work shown**. The early-finisher challenge is optional but turn it in too if you did it.

G08_MATH_Linear_01 — Linear Relationships in Four Forms Student Packet



TEACHER ANSWER KEY FOLLOWS

Do not copy the pages after this divider for students.

Answer Key: Linear Relationships in Four Forms

Grade: 8 **Subject:** Mathematics **For teacher use only** Math

How to use this key

Answers are grouped by section and question number, with full worked steps. Accept any correct method that reaches the right value — alternate strategies (using different table rows, counting up on the graph, or reasoning from the story) are noted where they apply. Watch-for notes flag common misconceptions. A basic calculator is allowed, so grade the **reasoning and steps**, not just the arithmetic. Estimated grading time: ~5–7 min per packet.

Start Warm-Up: Retrieval ~1 min

1 Evaluate the rule $y = 2x + 5$ at $x = 4$.

$2 \times 4 = 8$, then $8 + 5 = \mathbf{13}$. So $y = \mathbf{13}$. Full credit for showing the multiply-then-add order. *Watch-for:* a student who writes $2 \times (4 + 5) = 18$ added before multiplying — remind them to multiply mx first.

2 Read the point at $x = 2$ from Figure 1.

At $x = 2$, the marked point is **(2, 10)**. Accept the ordered pair (2, 10). *Alternate:* a student may also get this from $y = 3x + 4 \rightarrow 3(2) + 4 = 10$.

Build Study the Math ~1 min

3 Water depth at 6 minutes ($y = 3x + 4$).

$y = 3 \times 6 + 4 = 18 + 4 = \mathbf{22 \text{ inches}}$. *Alternate:* extend the table by adding 3 each minute ($16 \rightarrow 19 \rightarrow 22$). Both are correct.

Apply Use What You Know ~3 min

4 Match the four forms (gym membership, Table B).

4a. Using rows (2, 30) and (4, 40): $m = (40 - 30) \div (4 - 2) = 10 \div 2 = \mathbf{5 \text{ dollars per visit}}$. (Any two rows work: e.g., (0, 20) and (6, 50) $\rightarrow 30 \div 6 = 5$.)

4b. y-intercept $b = \mathbf{20}$ (the cost at $x = 0$ visits, read straight from the table). Equation: $y = 5x + 20$.

4c. The **slope** means the cost goes up **\$5 per visit** (each visit adds \$5). The **y-intercept** means the **\$20 sign-up fee** you pay before any visits (at 0 visits the cost is already \$20).

4d. It is **NOT proportional**, because the y-intercept is 20, not 0 — the line does not pass through the origin (0, 0). (A proportional relationship must start at 0.)

5 Error analysis: a mis-found slope for Table B.

The mistake: The student **reversed rise and run** — they divided **run** $\div$ **rise** instead of **rise** $\div$ **run**. Slope is the change in y (rise) over the change in x (run), so the differences are flipped.

Correct work: $m = \text{rise} \div \text{run} = (40 - 30) \div (4 - 2) = 10 \div 2 = \mathbf{5 \text{ dollars per visit}}$ (not 0.2).

Full credit requires (1) naming that they flipped rise and run (used run/rise), and (2) the correct slope of 5. *Sense-check to offer:* 0.2 dollars per visit would mean 20¢ a visit, which does not match the table jumping \$10 for every 2 visits.

6 Multi-step application (predict from $y = 5x + 20$).

6a. $y = 5(12) + 20 = 60 + 20 = \text{\$80}$ for 12 visits.

6b. Solve $5x + 20 = 75 \rightarrow$ subtract 20: $5x = 55 \rightarrow$ divide by 5: $x = \mathbf{11}$ visits. *Check:* $5(11) + 20 = 55 + 20 = 75$.

✓ *Alternate:* a student may count up from \$20 by \$5 steps until reaching \$75 (11 steps) — accept it.

Explain Justify Your Strategy (Q7) ~1–2 min

7 Bike rental for 3 hours: Plan A $y = 4x + 10$ vs. Plan B $y = 6x$.

Model answer. Claim: For a 3-hour rental, **Plan B** costs less; and **Plan B is the proportional one.** Evidence: Plan A: $y = 4(3) + 10 = 12 + 10 = \text{\$22}$. Plan B: $y = 6(3) = \text{\$18}$. $\$18 < \22 , so Plan B is cheaper for 3 hours.

Reasoning: Plan B has y-intercept $b = 0$, so it passes through the origin and is **proportional** ($y = 6x$). Plan A has y-intercept $b = 10$ (a \$10 sign-up), so it is **not proportional**. Even though Plan A's slope (\$4/hr) is smaller than Plan B's (\$6/hr), Plan A's \$10 head start makes it cost more at 3 hours.

Note: Full credit requires both computed costs (\$22 and \$18), naming Plan B as cheaper, and identifying Plan B as proportional ($b = 0$). *Extension worth noting:* the two plans are equal when $4x + 10 = 6x \rightarrow x = 5$ hours; beyond 5 hours Plan A becomes cheaper. Students are not required to find this, but it is a strong discussion point.

Justification scoring rubric (3 points)

Score	Claim	Evidence	Reasoning
3	Names Plan B as cheaper for 3 hours AND names Plan B as proportional.	Both costs computed correctly (\$22 and \$18) with the numbers shown.	Explains proportional means $b = 0$ and correctly links slope and y-intercept to the comparison.
2	Names Plan B as cheaper (proportional part missing or partly right).	One cost computed, or both with a minor arithmetic slip.	Reasoning present but incomplete or missing the y-intercept link.
1	A plan named with weak or no support.	Evidence largely missing or incorrect.	Little or flawed reasoning.
0	No/incorrect claim.	No evidence.	No reasoning.

Close ACE ~1 min

ACE Articulate / Connect / Extend.

Articulate: Accept any correct plain-language method: "pick two rows, subtract the y-values to get the rise, subtract the x-values in the same order to get the run, then divide rise $\div$ run."

Connect: Valid items where $y = mx + b$ was used include Q1 (as $y = 2x + 5$), Q3, Q4b, Q6, or Q7. Any of these earns credit if correctly named.

Extend: Any genuine non-proportional linear situation with a starting amount and a steady rate — e.g., a taxi with a base fare plus a per-mile rate, a phone plan with a monthly fee plus a per-gigabyte charge, savings that start at some amount and grow by a fixed weekly deposit. Must have a nonzero starting amount to count as non-proportional.

Challenge Early Finisher (optional)

EF Write a context for $y = 8x + 15$ (must be non-proportional).

(1) A sample situation: "A plumber charges a **\$15 service fee** to come out, plus **\$8 per hour** of work. Let x = hours worked and y = total charge in dollars." Many correct stories exist (a \$15 flat delivery fee plus \$8 per item; a gift card starting balance, etc.).

(2) Meaning: The **slope 8** is the **rate of \$8 per hour** (each hour adds \$8). The **y-intercept 15** is the **\$15 starting amount / fee** charged before any hours (the cost at $x = 0$).

(3) Predict y at $x = 5$: $y = 8(5) + 15 = 40 + 15 = \55 .

Full credit requires a clear x and y , a story with a real starting amount (so it is non-proportional), a correct interpretation of 8 and 15, and the correct value 55.

Watch Common misconceptions

- **Slope vs. y-intercept.** Students may swap the two — reporting the starting amount as the rate, or the rate as the starting amount. In $y = 5x + 20$, $m = 5$ is the per-visit rate and $b = 20$ is the sign-up fee. Anchor: the number multiplied by x is the slope; the number added on is the y-intercept (the value at $x = 0$).
- **Reversing rise and run.** The Q5 error: computing $\text{run} \div \text{rise}$ (giving 0.2) instead of $\text{rise} \div \text{run}$ (giving 5). Slope is **change in y over change in x** — always y -difference on top. Subtract the x -values and y -values in the **same order**.
- **Assuming all lines are proportional.** Students often treat any straight line as proportional. A line is proportional **only if it passes through (0, 0)**, i.e., $b = 0$. Figure 1 and Table B both have $b \neq 0$, so they are linear but **not** proportional (Q4d and Q7 check this).
- **Adding before multiplying.** In Q1/Q3/Q6, evaluate mx first, then add b — do not add x and b before multiplying.
- **Stopping at the wrong step when solving.** In Q6b, after $5x = 55$ a student may forget to divide by 5. Encourage a check by substituting the answer back into the equation.

Teacher follow-up based on likely errors

If many students miss Q5, do a quick 5-minute practice labeling rise and run on two or three graphs, always keeping the y -difference on top. If Q4d or Q7 show the "all lines are proportional" idea, re-anchor that only a line through (0, 0) is proportional ($b = 0$), and contrast it with a line that has a sign-up fee. The Q4c interpretations reveal who can attach meaning to slope and y-intercept in context — a good warm-up discussion next class. The Q7 justifications show who can compare two models using both slope and starting amount.