

Linear Relationships in Four Forms

Grade: 8 **Subject:** Mathematics **Time:** ~45 min core + ~15 min extension **Work mode:** Independent · pencil + packet Math

Start Here

This is a **paper math packet** about **linear relationships** — the kind that make a straight line. You will connect the same relationship shown four ways: a **table**, a **graph**, an **equation**, and a **real-world story**. Work by yourself with a pencil. **A calculator is not required, but a basic calculator is allowed** if you have one. **Show your work** — every answer should show the numbers you used, not just a final number. Everything you need (formulas, a worked example, a graph) is printed right here. If a question is hard, skip it, keep going, and come back. Circle or box your final answers.

Start Warm-Up: Retrieval 5 min

Bring back what you already know about rules and reading a graph.

1Evaluate a rule. A rule says $y = 2x + 5$. Find the value of y when $x = 4$. Show the multiplication and the addition you used.

2Read a point off a line. On the graph in the Build section (Figure 1), the line passes through several marked points. Find the point where $x = 2$ and write it as an ordered pair (x, y) . (You may look ahead to Figure 1 to answer this.)

Build Study the Math 5–10 min

What you need to know (with formulas)

A **linear relationship** between two quantities x and y makes a **straight line** when graphed. It changes by the **same amount** each step.

The **rate of change** (also called the **slope**, written **m**) tells how much y changes for each 1 that x increases. From two points (x_1, y_1) and (x_2, y_2) :

$$m = \text{rise} \div \text{run} = (y_2 - y_1) \div (x_2 - x_1)$$

The **y-intercept** (written **b**) is the value of y when $x = 0$ — the point **$(0, b)$** where the line crosses the vertical axis. Together they give the **slope-intercept form** of a line:

$$y = mx + b \quad (m = \text{slope}, b = \text{y-intercept})$$

A linear relationship can be shown **four ways**, and all four match the same m and b :

- **Table:** as x goes up by a fixed step, y goes up (or down) by a fixed step. $m = (\text{change in } y) \div (\text{change in } x)$. The row where $x = 0$ gives b .
- **Graph:** a straight line. It crosses the y -axis at $(0, b)$ and rises m units for each 1 unit right.
- **Equation:** $y = mx + b$.

- **Situation:** a real story with a **starting amount** (b) and a steady **rate** (m) per one.

Proportional vs. non-proportional. A linear relationship is **proportional** only when $b = 0$ — the line passes through the origin $(0, 0)$ and $y = mx$. If b is **not 0** (there is a starting amount), the line is **linear but NOT proportional**; it crosses the y -axis above or below the origin.

Tip: not every straight line is proportional. Check the y -intercept — only a line through $(0, 0)$ is a proportional relationship.

Worked Example — one relationship, four forms

A pool already holds some water, then a hose adds water at a steady rate. The depth starts at **4 inches** and rises **3 inches every minute**. Because there is a **starting amount of 4**, this is **linear but NOT proportional**.

Slope (rate of change): $m = 3$ inches per minute. **y-intercept:** $b = 4$ inches (the depth at 0 minutes).

Equation: $y = 3x + 4$, where x = minutes and y = depth in inches.

Check the slope from two table rows: using $(1, 7)$ and $(2, 10)$, $m = (10 - 7) \div (2 - 1) = 3 \div 1 = 3$. ✓

Table (each step of 1 in x adds 3 to y) and **graph** below. The line crosses the y -axis at $(0, 4)$ — **not** the origin — so it is non-proportional.

Water depth over time ($y = 3x + 4$).

Time x (minutes) Depth y (inches) Change in y

0	4	— (start)
1	7	+3
2	10	+3
3	13	+3
4	16	+3

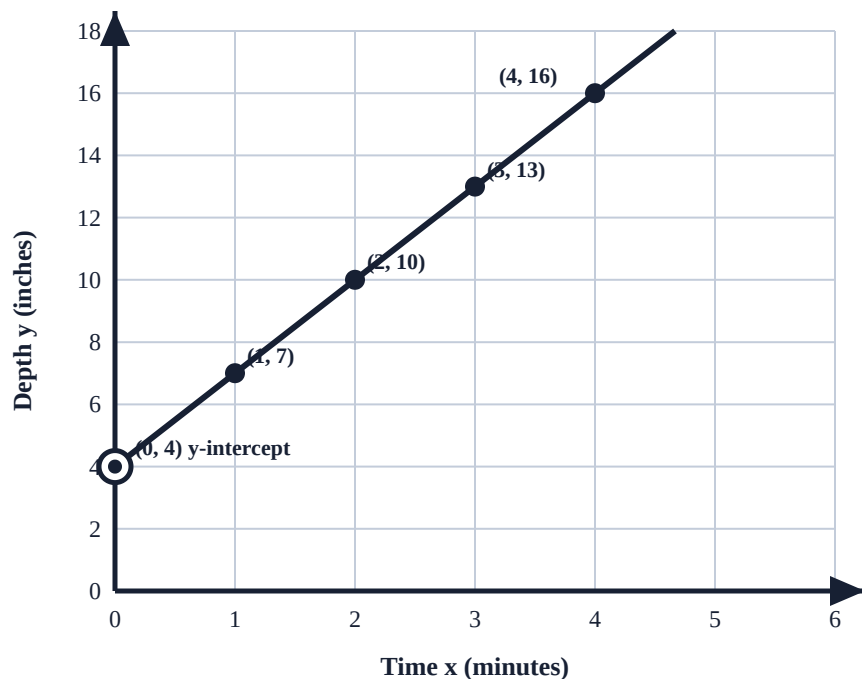


Figure 1. The graph of $y = 3x + 4$. It is a straight line that crosses the vertical axis at **(0, 4)** — not at the origin — so it is **linear but not proportional**. Each printed point (1, 7), (2, 10), (3, 13), (4, 16) matches a table row and rises 3 inches for every 1 minute. The line and points are drawn in black with text labels so the figure reads clearly in grayscale.

3Using the worked example, how deep would the water be at **6 minutes**? Use $y = 3x + 4$ and show your work.

Apply Use What You Know 20–25 min

Show the numbers you use in every item. A basic calculator is allowed.

4**Match the four forms.** A gym charges a one-time sign-up fee plus a fixed price for each visit. The **same** linear relationship is shown below as a table, a graph description, an equation, and a story. Study Table B, then answer 4a–4d.

Table B. Total cost of a gym membership (a linear relationship).

Visits x Total cost y (dollars)

0	20
2	30
4	40
6	50

Story: "A gym charges a one-time **\$20 sign-up fee**, then **\$5 for each visit**. Total cost = sign-up fee + \$5 times the number of visits."

4a. Find the **slope** m (dollars per visit). Use two rows of the table and show $m = (\text{change in } y) \div (\text{change in } x)$.

4b. Find the **y-intercept** b (the cost at 0 visits), then write the equation $y = mx + b$ for this relationship.

4c. **Interpret in context.** In this story, what does the **slope** mean, and what does the **y-intercept** mean? Use the words "per visit" and "sign-up fee."

4d. Is this relationship **proportional** or **not proportional**? Explain using the y-intercept in one sentence.

5**Error analysis (a mis-found slope).** A student found the slope for Table B and wrote the work shown. **Find the mistake, explain it, and give the correct slope.**

Student's work (using the rows (2, 30) and (4, 40)):

$$m = \text{run} \div \text{rise} = (4 - 2) \div (40 - 30)$$

$$m = 2 \div 10 = 0.2. \text{ So the slope is 0.2 dollars per visit.}$$

What is wrong, and what is the correct slope? Fix it in the box.

6**Multi-step application (predict a value from the model).** Use the gym relationship $y = 5x + 20$ from item 4.

6a. Predict the **total cost of 12 visits**. Show $y = 5(12) + 20$.

6b. A member paid a total of \$75. How many **visits** did they make? Set up $5x + 20 = 75$ and solve for x . Show your steps.

Explain Justify Your Strategy 5–10 min

Question 7. Two plans charge for renting a bike. **Plan A:** $y = 4x + 10$ (a \$10 sign-up plus \$4 per hour). **Plan B:** $y = 6x$ (just \$6 per hour, no sign-up). **For a 3-hour rental, which plan costs less, and why?** Also state which plan is **proportional**. Write a **Claim** (your answer), **Evidence** (the numbers you calculated), and **Reasoning** (why your strategy works, using slope and y-intercept).

Sentence stems you may use: "For 3 hours, Plan ____ costs less because..." · "I found Plan A's cost by... and Plan B's cost by..." · "Plan B is proportional because its y-intercept is..." · "The slope tells me... and the y-intercept tells me..."

Claim (your answer / strategy)

Evidence (the numbers you used)

Reasoning (why the strategy works)

Close ACE Wrap-Up 5 min

Articulate

In your own words, explain how to find the **slope** from two rows of a table using rise over run.

Connect

Point to one item in this packet where you used $y = mx + b$. Give its number.

Extend

Give a **new** real-life example of a **non-proportional** linear relationship (a starting amount plus a steady rate) that was *not* in this packet.

Continue Early Finisher optional · ~15 min

If you finish early (a challenge — no new materials needed):

Write a context for an equation. Here is an equation with no story: $y = 8x + 15$. On the back of this page: (1) invent a **real-world situation** that this equation could describe, clearly telling what x and y stand for; (2) explain what the **slope 8** and the **y-intercept 15** mean in your story; and (3) use your model to **predict y when $x = 5$** , showing your work. Make sure your story is **non-proportional** (it must have a starting amount).

Turn in: Hand in this whole packet with your name, class period, and date filled in. Make sure questions 1–7 and the ACE box are answered **with your work shown**. The early-finisher challenge is optional but turn it in too if you did it.

G08_MATH_Linear_01 — Linear Relationships in Four Forms Student Packet