

Answer Key: Linear Relationships in Four Forms

Grade: 8 **Subject:** Mathematics **For teacher use only** Math

How to use this key

Answers are grouped by section and question number, with full worked steps. Accept any correct method that reaches the right value — alternate strategies (using different table rows, counting up on the graph, or reasoning from the story) are noted where they apply. Watch-for notes flag common misconceptions. A basic calculator is allowed, so grade the **reasoning and steps**, not just the arithmetic. Estimated grading time: ~5–7 min per packet.

Start Warm-Up: Retrieval ~1 min

1 Evaluate the rule $y = 2x + 5$ at $x = 4$.

$2 \times 4 = 8$, then $8 + 5 = 13$. So $y = 13$. Full credit for showing the multiply-then-add order. *Watch-for:* a student who writes $2 \times (4 + 5) = 18$ added before multiplying — remind them to multiply mx first.

2 Read the point at $x = 2$ from Figure 1.

At $x = 2$, the marked point is **(2, 10)**. Accept the ordered pair (2, 10). *Alternate:* a student may also get this from $y = 3x + 4 \rightarrow 3(2) + 4 = 10$.

Build Study the Math ~1 min

3 Water depth at 6 minutes ($y = 3x + 4$).

$y = 3 \times 6 + 4 = 18 + 4 = 22$ **inches**. *Alternate:* extend the table by adding 3 each minute ($16 \rightarrow 19 \rightarrow 22$). Both are correct.

Apply Use What You Know ~3 min

4 Match the four forms (gym membership, Table B).

4a. Using rows (2, 30) and (4, 40): $m = (40 - 30) \div (4 - 2) = 10 \div 2 = 5$ **dollars per visit**. (Any two rows work: e.g., (0, 20) and (6, 50) $\rightarrow 30 \div 6 = 5$.)

4b. y-intercept $b = 20$ (the cost at $x = 0$ visits, read straight from the table). Equation: $y = 5x + 20$.

4c. The **slope** means the cost goes up **\$5 per visit** (each visit adds \$5). The **y-intercept** means the **\$20 sign-up fee** you pay before any visits (at 0 visits the cost is already \$20).

4d. It is **NOT proportional**, because the y-intercept is 20, not 0 — the line does not pass through the origin (0, 0). (A proportional relationship must start at 0.)

5 Error analysis: a mis-found slope for Table B.

The mistake: The student **reversed rise and run** — they divided **run** $\div$ **rise** instead of **rise** $\div$ **run**. Slope is the change in y (rise) over the change in x (run), so the differences are flipped.

Correct work: $m = \text{rise} \div \text{run} = (40 - 30) \div (4 - 2) = 10 \div 2 = 5$ **dollars per visit** (not 0.2).

Full credit requires (1) naming that they flipped rise and run (used run/rise), and (2) the correct slope of 5. *Sense-check to offer:* 0.2 dollars per visit would mean 20¢ a visit, which does not match the table jumping \$10 for every 2 visits.

6 Multi-step application (predict from $y = 5x + 20$).

6a. $y = 5(12) + 20 = 60 + 20 = \text{\$80}$ for 12 visits.

6b. Solve $5x + 20 = 75 \rightarrow$ subtract 20: $5x = 55 \rightarrow$ divide by 5: $x = \mathbf{11}$ visits. *Check:* $5(11) + 20 = 55 + 20 = 75$.

✓ *Alternate:* a student may count up from \$20 by \$5 steps until reaching \$75 (11 steps) — accept it.

Explain Justify Your Strategy (Q7) ~1–2 min

7 Bike rental for 3 hours: Plan A $y = 4x + 10$ vs. Plan B $y = 6x$.

Model answer. Claim: For a 3-hour rental, **Plan B** costs less; and **Plan B is the proportional one.** Evidence: Plan A: $y = 4(3) + 10 = 12 + 10 = \text{\$22}$. Plan B: $y = 6(3) = \text{\$18}$. $\$18 < \22 , so Plan B is cheaper for 3 hours.

Reasoning: Plan B has y-intercept $b = 0$, so it passes through the origin and is **proportional** ($y = 6x$). Plan A has y-intercept $b = 10$ (a \$10 sign-up), so it is **not proportional**. Even though Plan A's slope (\$4/hr) is smaller than Plan B's (\$6/hr), Plan A's \$10 head start makes it cost more at 3 hours.

Note: Full credit requires both computed costs (\$22 and \$18), naming Plan B as cheaper, and identifying Plan B as proportional ($b = 0$). *Extension worth noting:* the two plans are equal when $4x + 10 = 6x \rightarrow x = 5$ hours; beyond 5 hours Plan A becomes cheaper. Students are not required to find this, but it is a strong discussion point.

Justification scoring rubric (3 points)

Score	Claim	Evidence	Reasoning
3	Names Plan B as cheaper for 3 hours AND names Plan B as proportional.	Both costs computed correctly (\$22 and \$18) with the numbers shown.	Explains proportional means $b = 0$ and correctly links slope and y-intercept to the comparison.
2	Names Plan B as cheaper (proportional part missing or partly right).	One cost computed, or both with a minor arithmetic slip.	Reasoning present but incomplete or missing the y-intercept link.
1	A plan named with weak or no support.	Evidence largely missing or incorrect.	Little or flawed reasoning.
0	No/incorrect claim.	No evidence.	No reasoning.

Close ACE ~1 min

ACE Articulate / Connect / Extend.

Articulate: Accept any correct plain-language method: "pick two rows, subtract the y-values to get the rise, subtract the x-values in the same order to get the run, then divide rise $\div$ run."

Connect: Valid items where $y = mx + b$ was used include Q1 (as $y = 2x + 5$), Q3, Q4b, Q6, or Q7. Any of these earns credit if correctly named.

Extend: Any genuine non-proportional linear situation with a starting amount and a steady rate — e.g., a taxi with a base fare plus a per-mile rate, a phone plan with a monthly fee plus a per-gigabyte charge, savings that start at some amount and grow by a fixed weekly deposit. Must have a nonzero starting amount to count as non-proportional.

Challenge Early Finisher (optional)

EF Write a context for $y = 8x + 15$ (must be non-proportional).

(1) A sample situation: "A plumber charges a **\$15 service fee** to come out, plus **\$8 per hour** of work. Let x = hours worked and y = total charge in dollars." Many correct stories exist (a \$15 flat delivery fee plus \$8 per item; a gift card starting balance, etc.).

(2) Meaning: The **slope 8** is the **rate of \$8 per hour** (each hour adds \$8). The **y-intercept 15** is the **\$15 starting amount / fee** charged before any hours (the cost at $x = 0$).

(3) Predict y at $x = 5$: $y = 8(5) + 15 = 40 + 15 = \55 .

Full credit requires a clear x and y , a story with a real starting amount (so it is non-proportional), a correct interpretation of 8 and 15, and the correct value 55.

Watch Common misconceptions

- **Slope vs. y-intercept.** Students may swap the two — reporting the starting amount as the rate, or the rate as the starting amount. In $y = 5x + 20$, $m = 5$ is the per-visit rate and $b = 20$ is the sign-up fee. Anchor: the number multiplied by x is the slope; the number added on is the y-intercept (the value at $x = 0$).
- **Reversing rise and run.** The Q5 error: computing $\text{run} \div \text{rise}$ (giving 0.2) instead of $\text{rise} \div \text{run}$ (giving 5). Slope is **change in y over change in x** — always y -difference on top. Subtract the x -values and y -values in the **same order**.
- **Assuming all lines are proportional.** Students often treat any straight line as proportional. A line is proportional **only if it passes through (0, 0)**, i.e., $b = 0$. Figure 1 and Table B both have $b \neq 0$, so they are linear but **not** proportional (Q4d and Q7 check this).
- **Adding before multiplying.** In Q1/Q3/Q6, evaluate mx first, then add b — do not add x and b before multiplying.
- **Stopping at the wrong step when solving.** In Q6b, after $5x = 55$ a student may forget to divide by 5. Encourage a check by substituting the answer back into the equation.

Teacher follow-up based on likely errors

If many students miss Q5, do a quick 5-minute practice labeling rise and run on two or three graphs, always keeping the y -difference on top. If Q4d or Q7 show the "all lines are proportional" idea, re-anchor that only a line through (0, 0) is proportional ($b = 0$), and contrast it with a line that has a sign-up fee. The Q4c interpretations reveal who can attach meaning to slope and y-intercept in context — a good warm-up discussion next class. The Q7 justifications show who can compare two models using both slope and starting amount.