

Substitute Guide: Proportional Reasoning and Percents

Grade: 7 **Subject:** Mathematics **Time:** ~45 min core + ~15 min extension Math

Learning goal (plain language)

Students practice **unit rates** (a price or amount "per one"), **proportional relationships** (a steady rate shown as a table, a graph, and an equation $y = k \cdot x$), and **percents** (tax, tip, and discounts). **You do not need a math background to run this.** Every formula and a worked example are printed in the student packet. Students **show their work**; you do not need to teach the method — the packet does the teaching. Your job is to hand it out, read the start script, keep the room on task, and collect the packets.

Standards alignment

Framework: 19 TAC §111.27 (Grade 7 Mathematics). Knowledge & skills **7.4** — proportional relationships and percents.

- **Primary — 7.4A:** represent constant rates of change / the constant of proportionality ($k = y/x$) in tables, graphs, equations, and situations.
- **Primary — 7.4D:** solve problems involving ratios, rates, and percents, including tax, tip, and discount.
- **Supporting — 7.4E:** convert between measurement/rate representations to compare proportional situations.

Provisional — pending educator verification against the current official TAC source.

(Standards are paraphrased here, not quoted. Please confirm codes and wording against your official source.)

Setup Before class (5 min)

- Count and hand out one **student packet** per student.
- Confirm each student has a **pencil**. A **basic calculator is allowed but not required**; students who have one may use it.
- Write the date on the board and remind students to fill in the name line.
- Keep the **answer key** with you — it is a separate file and is **not** in the student packet.

Materials What's needed

- Printed student packet (4–6 pages) per student.
- Pencil per student; a basic four-function calculator is optional.
- This guide and the separate answer key (teacher only).
- No technology, no internet, no special supplies.

Timing Suggested pacing (~45–60 min)

Segment	Time	What students do
Start — Retrieval warm-up	5 min	Q1–Q2 (unit rate, simple percent)
Build — Study the math	5–10 min	Read reference box, worked example, graph; Q3
Apply — Use what you know	20–25 min	Q4–Q7 (four forms, tax/tip, error analysis, recipe)
Explain — Justify (CER)	5–10 min	Q8 claim/evidence/reasoning
Close — ACE	5 min	Articulate / Connect / Extend
Continue — early finishers	optional ~15 min	Two-discounts challenge

Script Read aloud to start

"Today's assignment is a paper math packet called *Proportional Reasoning and Percents*. You will work by yourself with a pencil. A basic calculator is allowed if you have one, but you do not need one. The most important rule: **show your work** — write down the numbers you use, not just a final answer. Everything you need is printed in the packet, including a worked example and all the formulas. Start with the warm-up, then read the Build section before the Apply questions. If a question is tricky, skip it and come back. Put your name, class period, and date at the top now. You have about 45 minutes; raise your hand if you need help reading a question."

Support When students ask for help

- **Students show their work; you do not need to teach the method.** Point them back to the printed **reference box** and the **worked example** — the method is right there.
- **You may read any part aloud** to a student or the class.
- If a student is stuck, prompt with "What does the worked example do?" or "What's the unit rate — the amount for one?" rather than giving the answer.
- For percents, remind them the packet shows: multiply for "percent of," and subtract for a discount ("percent off").
- It is fine if a student cannot finish the optional early-finisher challenge.

Access Accommodations & language support

- **Read-aloud** of any question is allowed for all students.
- Point students to the **worked example**, the **formulas** in the reference box, and the **sentence stems** for Q8 and ACE.
- A **calculator is allowed** for any student who wants one — this removes the arithmetic barrier so students can focus on the reasoning.
- Extended time is fine; the core can stop after Q8 if time runs short.
- Students may show work with numbers and short phrases if writing full sentences is a barrier, as long as the reasoning is clear.
- The packet is grayscale-safe — it prints and reads clearly in black and white.

Tools Allowed & not allowed

Allowed: pencil, the printed packet, a basic (four-function) calculator, quiet self-read-aloud. **Not needed / not allowed:** phones, internet, graphing/AI calculators, or getting answers from another student.

Collect at the end: Collect every packet, whether finished or not, with names on them. Stack them for the teacher. Note on a sticky how far most students got and any questions that caused confusion.

Answer key: The **answer key is a separate file** (answerkey.html) and is for the teacher only. Please do not hand it to students.

Teacher follow-up (for the returning teacher)

The highest-value items to review next class are the **error analysis (Q6)** and the **justification (Q8)**, which reveal the "percent of" vs. "percent off" confusion and whether students subtract a discount. Watch for **additive instead of multiplicative reasoning** on the recipe scaling (Q7). The four-forms item (Q4) shows who can move between a table, an equation, and a graph point using $k = y \div x$. The separate answer key lists worked steps, alternate strategies, and a 3-point rubric for Q8.

G07_MATH_Proportions_01 — Proportional Reasoning and Percents Substitute Guide

Proportional Reasoning and Percents

Grade: 7 **Subject:** Mathematics **Time:** ~45 min core + ~15 min extension **Work mode:** Independent · pencil + packet Math

Start Here

This is a **paper math packet** about unit rates, proportional relationships, and percents. Work by yourself with a pencil. **A calculator is not required, but a basic calculator is allowed** if you have one. **Show your work** — every answer should show the numbers you used, not just a final number. Everything you need (formulas, examples) is printed right here. If a question is hard, skip it, keep going, and come back. Circle or box your final answers.

Start Warm-Up: Retrieval 5 min

Bring back what you already know about rates and percents.

1Unit rate. A grocery store sells 3 pounds of apples for \$6.00. What is the price for **1 pound** (the unit rate in dollars per pound)? Show the division you used.

2Simple percent. A class has 25 students, and 20 of them ride the bus. What **percent** of the class rides the bus? Show your work.

Build Study the Math 5–10 min

What you need to know (with formulas)

Unit rate is a rate written as an amount "per 1." To find it, divide the two quantities. Example: \$6 for 3 lb $\rightarrow$ \$6 $\div$ 3 lb = **\$2 per pound**.

A **proportional relationship** connects two quantities x and y that always share the **same ratio**. The number you multiply x by to get y is the **constant of proportionality**, written **k** :

$$k = y \div x \rightarrow y = k \cdot x$$

The **same relationship** can be shown four ways, and they all give the same k :

- **Table:** every row's $y \div x$ equals k .
- **Graph:** a straight line through the point **(0, 0)** (the origin). The point $(1, k)$ is on the line.
- **Equation:** $y = k \cdot x$.
- **Situation:** a real story with a constant "per one" rate.

Percent means "out of 100," so a percent is a fraction over 100 ($25\% = 25/100 = 0.25$).

- **Percent of a number:** multiply. *percent (as a decimal) $\times$ whole*. Example: 15% of 40 = $0.15 \times 40 = 6$.
- **Percent increase:** new = whole + (percent $\times$ whole). A 20% tip on \$10 = $\$10 + (0.20 \times \$10) = \$12$.
- **Percent decrease:** new = whole – (percent $\times$ whole). A 20% discount on \$10 = $\$10 - (0.20 \times \$10) = \$8$.

Tip: "percent **of**" asks for the part; "percent **off**" (a discount) asks you to subtract the part from the original.

Worked Example — one relationship, four forms

A water dispenser fills cups at a steady rate. In **4 seconds** it pours **10 ounces**. Because it is steady, this is proportional.

Find k (ounces per second): $k = y \div x = 10 \text{ oz} \div 4 \text{ s} = 2.5 \text{ oz per second}$.

Equation: $y = 2.5x$, where x = seconds and y = ounces.

Table (each $y \div x = 2.5$) and **graph** below. The line goes through (0, 0), and the point (1, 2.5) shows the unit rate.

Water poured over time ($y = 2.5x$).

Time x (seconds)	Water y (ounces)	$y \div x$
0	0	—
2	5	2.5
4	10	2.5
6	15	2.5

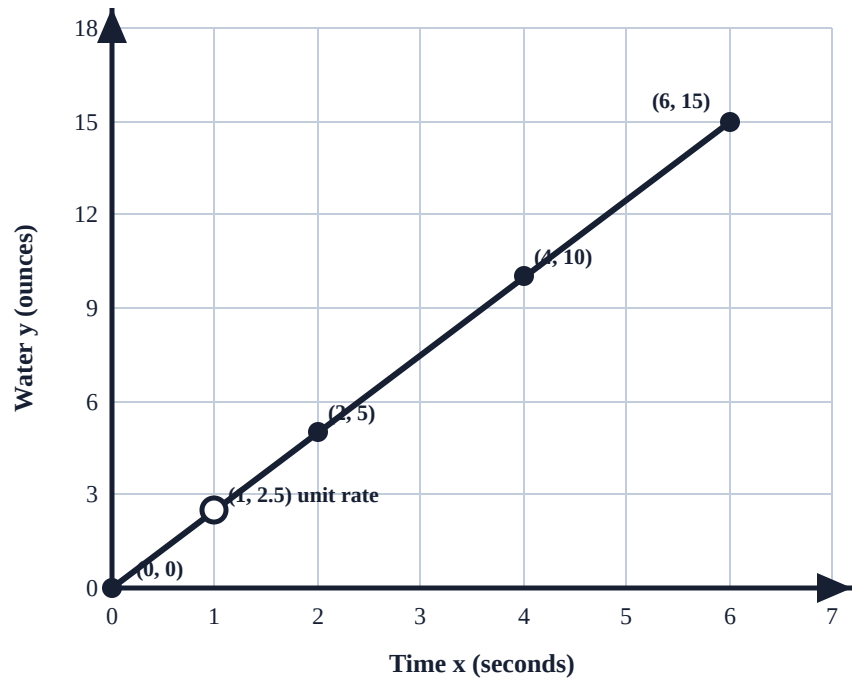


Figure 1. The graph of $y = 2.5x$. It is a straight line through the origin (0, 0). Each printed point (2, 5), (4, 10), (6, 15) matches a table row, and the hollow point (1, 2.5) shows the unit rate — the ounces poured in one second. The line and points are drawn in black so the figure reads clearly in grayscale.

3Using the worked example, how many ounces would the dispenser pour in **8 seconds**? Use $y = 2.5x$ and show your work.

Apply Use What You Know 20–25 min

Show the numbers you use in every item. A basic calculator is allowed.

4Match the four forms. A bakery sells muffins at a steady price. The relationship is described four ways below, but one form is missing a value and one situation must be matched. Study Table A, then answer 4a–4c.

Table A. Cost of muffins (a proportional relationship).

Muffins x Cost y (dollars)

2	3.00
4	6.00
6	9.00
10	?

4a. Find the constant of proportionality k (dollars per muffin). Show $y \div x$.

4b. Write the equation for cost y in terms of muffins x , then fill in the missing cost for 10 muffins.

4c. Which graph point would be on this line: (1, 1.50) or (1, 3.00)? Explain in one sentence why.

5Real-world percent (tax and tip). A family's restaurant bill is **\$40.00**. Sales tax is **8%**, and they want to leave a **15% tip** (the tip is figured on the \$40 food amount, before tax).

5a. How much is the 8% tax? Show 0.08×40 .

5b. How much is the 15% tip on the \$40 food? Show your work.

5c. What is the **total** the family pays (food + tax + tip)?

6Error analysis. A student solved this problem: "A \$50 jacket is marked **30% off**. What is the sale price?" Their work is shown in the box. **Find the mistake, explain it, and give the correct sale price.**

Student's work:

$$30\% \text{ of } \$50 = 0.30 \times 50 = \$15.$$

So the sale price is **\$15**.

What is wrong, and what is the correct sale price?

7Multi-step application (scaling a recipe). A muffin recipe makes **12 muffins** and uses **3 cups of flour** and **2 cups of sugar**. A baker wants to make **30 muffins**, keeping the same proportions.

7a. Find the flour needed per muffin (the unit rate), then the flour for 30 muffins.

7b. How many cups of sugar are needed for 30 muffins? Show your work.

Explain Justify Your Strategy 5–10 min

Question 8. A phone case costs \$20. Store A takes **25% off**. Store B takes **\$4 off**. **Which store gives the lower price, and why?** Write a **Claim** (your answer), **Evidence** (the numbers you calculated), and **Reasoning** (why your strategy works).

Sentence stems you may use: "The lower price is at Store ____ because..." · "I found Store A's price by... and Store B's price by..." · "25% of \$20 is... so Store A costs..." · "This strategy works because a percent means..."

Claim (your answer / strategy)

Evidence (the numbers you used)

Reasoning (why the strategy works)

Close ACE Wrap-Up 5 min

Articulate

In your own words, explain how to find the **constant of proportionality k** from a table.

Connect

Point to one item in this packet where you used $k = y \div x$ or a unit rate. Give its number.

Extend

Give a **new** real-life example of a proportional relationship (a steady "per one" rate) that was *not* in this packet.

Continue Early Finisher optional · ~15 min

If you finish early (a challenge — no new materials needed):

Two discounts in a row. A \$60 pair of shoes is first marked **20% off**. At the register, a coupon takes an **extra 10% off the already-reduced price**. On the back of this page: (1) find the final price step by step; (2) explain why the total discount is **not** 30% off the original \$60; and (3) find the single percent off the original \$60 that would give the same final price. Show all your work.

Turn in: Hand in this whole packet with your name, class period, and date filled in. Make sure questions 1–8 and the ACE box are answered **with your work shown**. The early-finisher challenge is optional but turn it in too if you did it.

G07_MATH_Proportions_01 — Proportional Reasoning and Percents Student Packet



TEACHER ANSWER KEY FOLLOWS

Do not copy the pages after this divider for students.

Answer Key: Proportional Reasoning and Percents

Grade: 7 **Subject:** Mathematics **For teacher use only** Math

How to use this key

Answers are grouped by section and question number, with full worked steps. Accept any correct method that reaches the right value — alternate strategies (scale factors, fraction ratios, or unit rates) are noted where they apply. Watch-for notes flag common misconceptions. A basic calculator is allowed, so grade the **reasoning and steps**, not just the arithmetic. Estimated grading time: ~5–7 min per packet.

Start Warm-Up: Retrieval ~1 min

1 Unit rate: 3 lb of apples for \$6.00.

$\$6.00 \div 3 \text{ lb} = \mathbf{\$2.00 \text{ per pound}}$. Full credit for showing the division and the "per pound" unit. *Alternate:* a ratio table halving to 1.5 lb then to 1 lb is fine if it lands on \$2/lb.

2 Percent: 20 of 25 students ride the bus.

$20 \div 25 = 0.8 = \mathbf{80\%}$. *Alternate:* scale the fraction 20/25 to /100 by multiplying by 4/4 $\rightarrow 80/100 = 80\%$. Accept either method.

Build Study the Math ~1 min

3 Ounces poured in 8 seconds ($y = 2.5x$).

$y = 2.5 \times 8 = \mathbf{20 \text{ ounces}}$. *Alternate:* 4 s $\rightarrow$ 10 oz, so 8 s is double $\rightarrow$ 20 oz (multiplicative reasoning). Both are correct.

Apply Use What You Know ~2–3 min

4 Match the four forms (muffins, Table A).

4a. $k = y \div x = 3.00 \div 2 = \mathbf{\$1.50 \text{ per muffin}}$ (check: $6 \div 4 = 1.50$; $9 \div 6 = 1.50$ — constant, so it is proportional).

4b. Equation: $y = \mathbf{1.50x}$. For 10 muffins: $y = 1.50 \times 10 = \mathbf{\$15.00}$.

4c. The point **(1, 1.50)** is on the line, because at $x = 1$ muffin the cost is the unit rate $k = \$1.50$. (0, 0) and (1, k) always lie on a proportional line; (1, 3.00) would mean \$3.00 per muffin, which does not match the table.

5 Restaurant bill: \$40 food, 8% tax, 15% tip on food.

5a. Tax = $0.08 \times 40 = \mathbf{\$3.20}$.

5b. Tip = $0.15 \times 40 = \mathbf{\$6.00}$.

5c. Total = $40 + 3.20 + 6.00 = \mathbf{\$49.20}$.

Note: The problem states the tip is on the \$40 food (pre-tax). If a student figures the 15% tip on \$43.20 (food + tax) they get $0.15 \times 43.20 = \$6.48$ and a total of \$49.68 — mark this as a reasonable real-world alternate **only if** they state that assumption; otherwise the intended answer is \$49.20.

6 Error analysis: \$50 jacket, 30% off.

The mistake: The student found the **discount amount** (\$15) but stopped there and called it the sale price. "30% off" means you must **subtract** the discount from the original price.

Correct work: Discount = $0.30 \times 50 = \$15$. Sale price = $50 - 15 = \$35$.

Alternate correct method: paying $100\% - 30\% = 70\%$ of the price $\rightarrow 0.70 \times 50 = \35 . Accept either as the "fix." Full credit requires (1) naming that they used "percent of" as the answer instead of subtracting, and (2) the correct \$35.

7 Scaling a recipe (12 muffins: 3 cups flour, 2 cups sugar) up to 30.

7a. Flour per muffin = $3 \div 12 = 0.25$ cup per muffin. For 30 muffins: $0.25 \times 30 = 7.5$ cups of flour.

7b. Sugar per muffin = $2 \div 12 = 1/6$ cup ≈ 0.1667 cup. For 30 muffins: $(1/6) \times 30 = 5$ cups of sugar.

Alternate (scale factor): $30 \div 12 = 2.5$, so multiply each amount by 2.5: flour $3 \times 2.5 = 7.5$ cups; sugar $2 \times 2.5 = 5$ cups. This is often the cleaner method — accept it fully.

Explain Justify Your Strategy (Q8) ~1–2 min

8 \$20 case: Store A 25% off vs. Store B \$4 off — which is lower?

Model answer. Claim: Store A gives the lower price. Evidence: Store A: 25% of \$20 = $0.25 \times 20 = \$5$ off, so $\$20 - \$5 = \$15$. Store B: $\$20 - \$4 = \$16$. Reasoning: A percent means "out of 100," so 25% off a \$20 item is one-quarter of \$20, which is \$5 — larger than Store B's flat \$4 off. A bigger discount leaves a lower price, so Store A (\$15) beats Store B (\$16).

Note: Full credit requires both computed prices (\$15 and \$16) and a reason that connects the percent to the dollar amount. A student who only says "25% is more than \$4" without computing the \$5 discount earns partial credit.

Justification scoring rubric (3 points)

Score	Claim	Evidence	Reasoning
3	Correct store named (Store A).	Both prices computed correctly (\$15 and \$16) with the numbers shown.	Clearly explains why 25% off (\$5) beats a flat \$4 off, so the strategy is sound.
2	Correct store named.	One price computed, or both with a minor arithmetic slip.	Reasoning present but incomplete or missing the percent-to-dollars link.
1	Store named with weak or no support.	Evidence largely missing or incorrect.	Little or flawed reasoning.
0	No/incorrect claim.	No evidence.	No reasoning.

Close ACE ~1 min

ACE Articulate / Connect / Extend.

Articulate: Accept any correct plain-language method: "pick a row and divide y by x; if every row gives the same number, that number is k."

Connect: Valid items where $k = y \div x$ or a unit rate was used include Q1, Q3, Q4a, or Q7a. Any of these earns credit if correctly named.

Extend: Any genuine constant "per one" rate not in the packet — e.g., miles per hour at steady speed, cost per gallon of gas, pages read per minute, dollars earned per hour. More than one answer is defensible.

Challenge Early Finisher (optional)

EF Two discounts in a row: \$60 shoes, 20% off then extra 10% off.

(1) Step by step: After 20% off: $60 - (0.20 \times 60) = 60 - 12 = \48 . Then 10% off the reduced price: $48 - (0.10 \times 48) = 48 - 4.80 = \43.20 final. (Or: $60 \times 0.80 \times 0.90 = \43.20 .)

(2) Why it is not 30% off: The second 10% is taken off **\$48**, not the original \$60, so it removes only \$4.80 instead of \$6. Percents do not simply add when they are applied to different bases (a multiplicative, not additive, situation).

(3) Single equivalent percent: $\text{Final} \div \text{original} = 43.20 \div 60 = 0.72$, so the shopper pays 72% of the price, which is **28% off** the original \$60 ($0.80 \times 0.90 = 0.72$). Accept 28%.

Watch Common misconceptions

- **Additive vs. multiplicative reasoning.** Students may "add up" instead of scaling. In Q7, going 12 $\rightarrow$ 30 muffins is $\times 2.5$, not "+18," so amounts scale by 2.5, not by adding 18. Reinforce the unit rate or scale factor.
- **"Percent of" vs. "percent off."** The Q6 error: finding 30% *of* \$50 (\$15) and reporting it as the sale price instead of subtracting it (sale = \$35). "Off" means subtract the part from the whole.
- **Reversing the base.** In Q5, the tip is on the \$40 food, not the after-tax total; in the early finisher, the second discount's base is the reduced \$48, not \$60. Always ask "percent *of what?*"
- **Non-proportional slips.** A relationship is only proportional if every row's $y \div x$ is the same and the graph passes through (0, 0). Students sometimes assume any table is proportional (Q4 checks this).
- **Stopping at the discount.** Reporting the discount amount as the price (Q6, Q8) is common; always confirm whether the question wants the part or the new total.

Teacher follow-up based on likely errors

If many students miss Q6, do a quick 5-minute sort of "percent of" vs. "percent off" word problems. If Q7 shows additive reasoning (answers like $3 + 18$), model the $\times 2.5$ scale factor and the unit-rate method side by side. If Q4c is missed, re-anchor that every proportional graph passes through (0, 0) and (1, k). The Q8 justifications reveal who can link a percent to an actual dollar amount — a good discussion starter next class.