

Answer Key: Proportional Reasoning and Percents

Grade: 7 **Subject:** Mathematics **For teacher use only** Math

How to use this key

Answers are grouped by section and question number, with full worked steps. Accept any correct method that reaches the right value — alternate strategies (scale factors, fraction ratios, or unit rates) are noted where they apply. Watch-for notes flag common misconceptions. A basic calculator is allowed, so grade the **reasoning and steps**, not just the arithmetic. Estimated grading time: ~5–7 min per packet.

Start Warm-Up: Retrieval ~1 min

1 Unit rate: 3 lb of apples for \$6.00.

$\$6.00 \div 3 \text{ lb} = \mathbf{\$2.00 \text{ per pound}}$. Full credit for showing the division and the "per pound" unit. *Alternate:* a ratio table halving to 1.5 lb then to 1 lb is fine if it lands on \$2/lb.

2 Percent: 20 of 25 students ride the bus.

$20 \div 25 = 0.8 = \mathbf{80\%}$. *Alternate:* scale the fraction 20/25 to /100 by multiplying by 4/4 $\rightarrow 80/100 = 80\%$. Accept either method.

Build Study the Math ~1 min

3 Ounces poured in 8 seconds ($y = 2.5x$).

$y = 2.5 \times 8 = \mathbf{20 \text{ ounces}}$. *Alternate:* 4 s $\rightarrow$ 10 oz, so 8 s is double $\rightarrow$ 20 oz (multiplicative reasoning). Both are correct.

Apply Use What You Know ~2–3 min

4 Match the four forms (muffins, Table A).

4a. $k = y \div x = 3.00 \div 2 = \mathbf{\$1.50 \text{ per muffin}}$ (check: $6 \div 4 = 1.50$; $9 \div 6 = 1.50$ — constant, so it is proportional).

4b. Equation: $y = \mathbf{1.50x}$. For 10 muffins: $y = 1.50 \times 10 = \mathbf{\$15.00}$.

4c. The point **(1, 1.50)** is on the line, because at $x = 1$ muffin the cost is the unit rate $k = \$1.50$. (0, 0) and (1, k) always lie on a proportional line; (1, 3.00) would mean \$3.00 per muffin, which does not match the table.

5 Restaurant bill: \$40 food, 8% tax, 15% tip on food.

5a. Tax = $0.08 \times 40 = \mathbf{\$3.20}$.

5b. Tip = $0.15 \times 40 = \mathbf{\$6.00}$.

5c. Total = $40 + 3.20 + 6.00 = \mathbf{\$49.20}$.

Note: The problem states the tip is on the \$40 food (pre-tax). If a student figures the 15% tip on \$43.20 (food + tax) they get $0.15 \times 43.20 = \$6.48$ and a total of \$49.68 — mark this as a reasonable real-world alternate **only if** they state that assumption; otherwise the intended answer is \$49.20.

6 Error analysis: \$50 jacket, 30% off.

The mistake: The student found the **discount amount** (\$15) but stopped there and called it the sale price. "30% off" means you must **subtract** the discount from the original price.

Correct work: Discount = $0.30 \times 50 = \$15$. Sale price = $50 - 15 = \$35$.

Alternate correct method: paying $100\% - 30\% = 70\%$ of the price $\rightarrow 0.70 \times 50 = \35 . Accept either as the "fix." Full credit requires (1) naming that they used "percent of" as the answer instead of subtracting, and (2) the correct \$35.

7 Scaling a recipe (12 muffins: 3 cups flour, 2 cups sugar) up to 30.

7a. Flour per muffin = $3 \div 12 = 0.25$ cup per muffin. For 30 muffins: $0.25 \times 30 = 7.5$ cups of flour.

7b. Sugar per muffin = $2 \div 12 = 1/6$ cup ≈ 0.1667 cup. For 30 muffins: $(1/6) \times 30 = 5$ cups of sugar.

Alternate (scale factor): $30 \div 12 = 2.5$, so multiply each amount by 2.5: flour $3 \times 2.5 = 7.5$ cups; sugar $2 \times 2.5 = 5$ cups. This is often the cleaner method — accept it fully.

Explain Justify Your Strategy (Q8) ~1–2 min

8 \$20 case: Store A 25% off vs. Store B \$4 off — which is lower?

Model answer. Claim: Store A gives the lower price. Evidence: Store A: 25% of \$20 = $0.25 \times 20 = \$5$ off, so $\$20 - \$5 = \$15$. Store B: $\$20 - \$4 = \$16$. Reasoning: A percent means "out of 100," so 25% off a \$20 item is one-quarter of \$20, which is \$5 — larger than Store B's flat \$4 off. A bigger discount leaves a lower price, so Store A (\$15) beats Store B (\$16).

Note: Full credit requires both computed prices (\$15 and \$16) and a reason that connects the percent to the dollar amount. A student who only says "25% is more than \$4" without computing the \$5 discount earns partial credit.

Justification scoring rubric (3 points)

Score	Claim	Evidence	Reasoning
3	Correct store named (Store A).	Both prices computed correctly (\$15 and \$16) with the numbers shown.	Clearly explains why 25% off (\$5) beats a flat \$4 off, so the strategy is sound.
2	Correct store named.	One price computed, or both with a minor arithmetic slip.	Reasoning present but incomplete or missing the percent-to-dollars link.
1	Store named with weak or no support.	Evidence largely missing or incorrect.	Little or flawed reasoning.
0	No/incorrect claim.	No evidence.	No reasoning.

Close ACE ~1 min

ACE Articulate / Connect / Extend.

Articulate: Accept any correct plain-language method: "pick a row and divide y by x; if every row gives the same number, that number is k."

Connect: Valid items where $k = y \div x$ or a unit rate was used include Q1, Q3, Q4a, or Q7a. Any of these earns credit if correctly named.

Extend: Any genuine constant "per one" rate not in the packet — e.g., miles per hour at steady speed, cost per gallon of gas, pages read per minute, dollars earned per hour. More than one answer is defensible.

Challenge Early Finisher (optional)

EF Two discounts in a row: \$60 shoes, 20% off then extra 10% off.

(1) Step by step: After 20% off: $60 - (0.20 \times 60) = 60 - 12 = \48 . Then 10% off the reduced price: $48 - (0.10 \times 48) = 48 - 4.80 = \43.20 **final**. (Or: $60 \times 0.80 \times 0.90 = \43.20 .)

(2) Why it is not 30% off: The second 10% is taken off **\$48**, not the original \$60, so it removes only \$4.80 instead of \$6. Percents do not simply add when they are applied to different bases (a multiplicative, not additive, situation).

(3) Single equivalent percent: $\text{Final} \div \text{original} = 43.20 \div 60 = 0.72$, so the shopper pays 72% of the price, which is **28% off** the original \$60 ($0.80 \times 0.90 = 0.72$). Accept 28%.

Watch Common misconceptions

- **Additive vs. multiplicative reasoning.** Students may "add up" instead of scaling. In Q7, going 12 $\rightarrow$ 30 muffins is $\times 2.5$, not "+18," so amounts scale by 2.5, not by adding 18. Reinforce the unit rate or scale factor.
- **"Percent of" vs. "percent off."** The Q6 error: finding 30% *of* \$50 (\$15) and reporting it as the sale price instead of subtracting it (sale = \$35). "Off" means subtract the part from the whole.
- **Reversing the base.** In Q5, the tip is on the \$40 food, not the after-tax total; in the early finisher, the second discount's base is the reduced \$48, not \$60. Always ask "percent *of what?*"
- **Non-proportional slips.** A relationship is only proportional if every row's $y \div x$ is the same and the graph passes through (0, 0). Students sometimes assume any table is proportional (Q4 checks this).
- **Stopping at the discount.** Reporting the discount amount as the price (Q6, Q8) is common; always confirm whether the question wants the part or the new total.

Teacher follow-up based on likely errors

If many students miss Q6, do a quick 5-minute sort of "percent of" vs. "percent off" word problems. If Q7 shows additive reasoning (answers like $3 + 18$), model the $\times 2.5$ scale factor and the unit-rate method side by side. If Q4c is missed, re-anchor that every proportional graph passes through (0, 0) and (1, k). The Q8 justifications reveal who can link a percent to an actual dollar amount — a good discussion starter next class.